On some equations linked to the meaning of DN Constant. New possible mathematical connections with the Ramanujan Recurring Numbers and some parameters of Number Theory, Theoretical Cosmology and String Theory

Michele Nardelli[1], Antonio Nardelli

Abstract

In this paper, we analyze some equations linked to the meaning of DN Constant. We describe new possible mathematical connections with the Ramanujan Recurring Numbers and some parameters of Number Theory, Theoretical Cosmology and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan

(1887 - 1920)

Introduction

In this paper, an octahedron could serve as a mathematical or conceptual model of the universe in the quantic phase, while the spherical surface could be used to describe the geometry of the bubble-universe.

The values $(2\sqrt{2})/\pi$, the golden ratio φ, $\zeta(2)$ and π, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$ the so called DN Constant:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe

bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

The above values $(2\sqrt{2})/\pi$, the golden ratio φ and π, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

The various mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe.

Regarding the fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

Proposal:

The initial octahedron: Let's imagine a regular octahedron, with perfectly symmetrical faces. Each face represents an ideal symmetry.

The emerging sphere: Inside the octahedron, there is an inscribed sphere. This sphere represents the bubble of the universe that emerges from the perturbations of the quantum vacuum during eternal inflation.

Expansion and transitions: As time passes, the universe expands. The faces of the octahedron begin to break, symbolizing "symmetry breaks." The sphere continues to grow, representing the expanding universe.

Constants and numbers: We integrate the mathematical results you obtained. For example, the golden ratio (φ) could be represented by a proportion between the dimensions of the octahedron and the sphere.

Entropy and complexity: Entropy increases as the universe evolves. We can represent this with a disordered growth of structures within the emerging sphere. Imagine this scene as an abstract work of art, where geometric shapes and cosmological concepts merge

In Fig.1 and Fig.2 let's imagine a regular octahedron representing the universe in its phase of high symmetry and very low entropy. Inside the octahedron we have an inscribed sphere that emerges from perturbations of the quantum vacuum during eternal inflation. As time passes, the universe expands, the faces of the octahedron break (symmetry breaks), and entropy increases. Spheres emerge from the octahedra, symbolizing the transition phases from a regime of very low entropy to a universe in which, with the passage of time, entropy increases, increasing the complexity of the universe itself.

Fig. 1

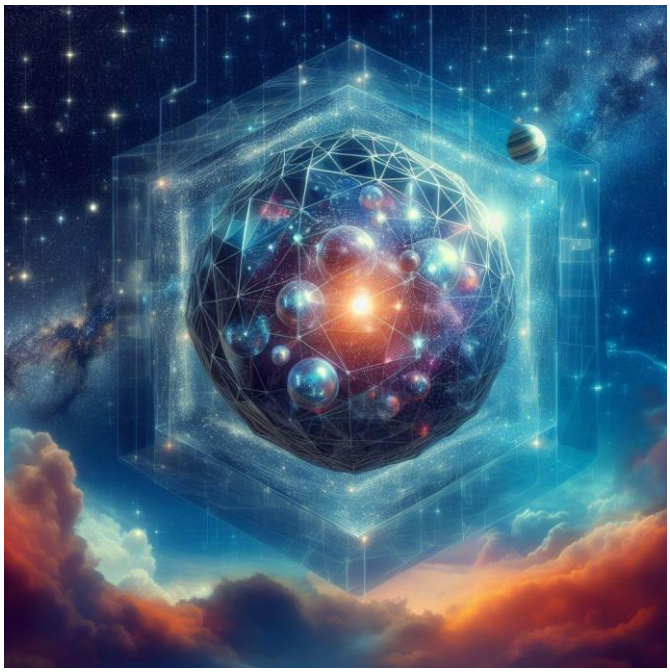
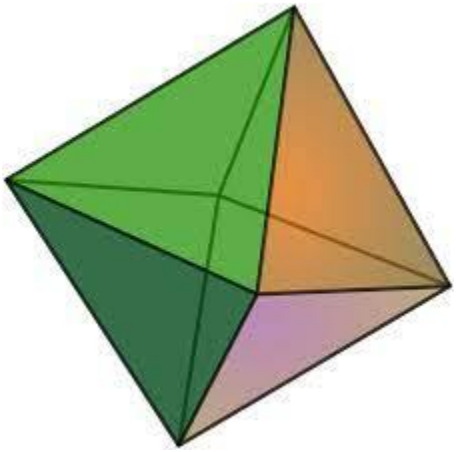
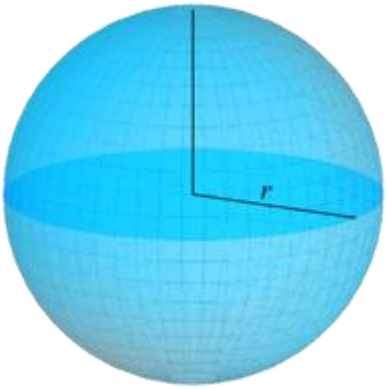


Fig. 2

Now, we have that:



Octahedron



Sphere

From the octahedron volume V = 1/3*√2 l^3 and, from the sphere volume, V = (4/3*π*r^3) , we consider the following relationship, for r = x:

4/3*π*x^3 = 1/3*√2*l^3

Input

$$\frac{4}{3}\pi x^3 = \frac{1}{3}\sqrt{2}\, l^3$$

Exact result

$$\frac{4\pi x^3}{3} = \frac{\sqrt{2}\, l^3}{3}$$

Alternate forms

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$\frac{4\pi x^3}{3} - \frac{\sqrt{2}\, l^3}{3} = 0$$

Real solution

$$x = \frac{(-1)^{2/3} l}{\sqrt{2}\, \sqrt[3]{\pi}}$$

Solutions

$$x = -\frac{\sqrt[3]{-\frac{1}{\pi}}\, l}{\sqrt{2}}$$

$$x = \frac{l}{\sqrt{2}\, \sqrt[3]{\pi}}$$

$$x = \frac{(-1)^{2/3} l}{\sqrt{2}\, \sqrt[3]{\pi}}$$

Integer solution

$l = 0, \quad x = 0$

Implicit derivatives

$$\frac{\partial x(l)}{\partial l} = \frac{l^2}{2\sqrt{2}\, \pi x^2}$$

$$\frac{\partial l(x)}{\partial x} = \frac{2\sqrt{2}\, \pi x^2}{l^2}$$

From the alternate form

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

for l = 8, we have that:

8/(sqrt(2) π^(1/3)) = 8/(2sqrt2 * Pi)^1/3

Input

$$\frac{8}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{8}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Result

True

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Thence:

l/(sqrt(2) π^(1/3)) = l/(2sqrt2 * Pi)^1/3

Input

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base-b logarithm

Now, we have that:

l/(2 sqrt(2) π)^(1/3) = (2sqrt2)/Pi

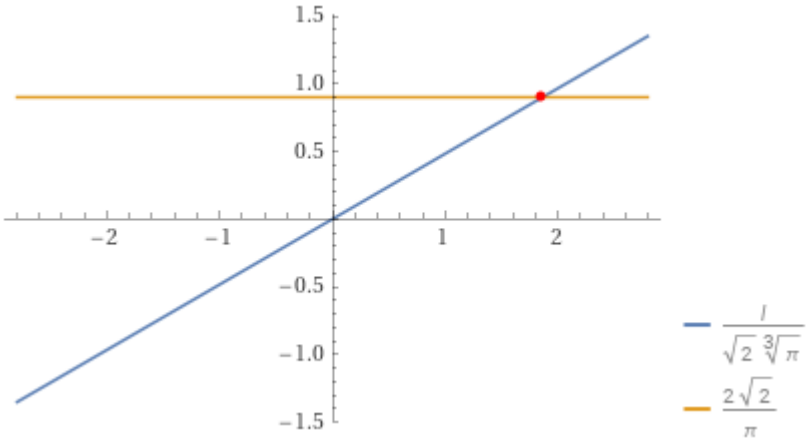
Input

$$\frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{2\sqrt{2}}{\pi}$$

Plot



Solution

$$l = \frac{4}{\pi^{2/3}}$$

$$l = \frac{4}{\sqrt[3]{\pi^2}}$$

Proposed physical meaning of DN Constant

From

GS 2.7b Griffiths 3rd edition, quantum mechanics, Problem 2.7 Part 2, Schrodinger equation - https://www.youtube.com/watch?v=gDvJwoeCIZM&t=2s

We analyze the following equation: (solution of the Schrodinger equation for a particle in a box of side proportional to *a*)

$$\psi(x,t) = \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar} \qquad c_n = \int_{-\infty}^{\infty} \psi(x,0)\psi(x)\,dx$$

$$c_n = \frac{\sqrt{6}}{a}\sqrt{\frac{2}{a}}\sqrt{\frac{2}{a}}\left[\int_0^{a/2} x\sin\left(\frac{n\pi}{a}x\right)dx + \int_{a/2}^{a}(a-x)\sin\left(\frac{n\pi}{a}x\right)dx\right]$$

$$c_n = \frac{2\sqrt{6}}{a^2}\left[\int_0^{a/2} x\sin\left(\frac{n\pi}{a}x\right)dx - \int_{a/2}^{a} x\sin\left(\frac{n\pi}{a}x\right)dx + a\int_{a/2}^{a}\sin\left(\frac{n\pi}{a}x\right)dx\right]$$

$$c_n = \frac{2\sqrt{6}}{a^2}[I_1 - I_2 + I_3]$$

$$c_n = \frac{2\sqrt{6}}{a^2}\left[\int_0^{a/2} x\sin\left(\frac{n\pi}{a}x\right)dx - \int_{a/2}^{a} x\sin\left(\frac{n\pi}{a}x\right)dx + a\int_{a/2}^{a}\sin\left(\frac{n\pi}{a}x\right)dx\right]$$

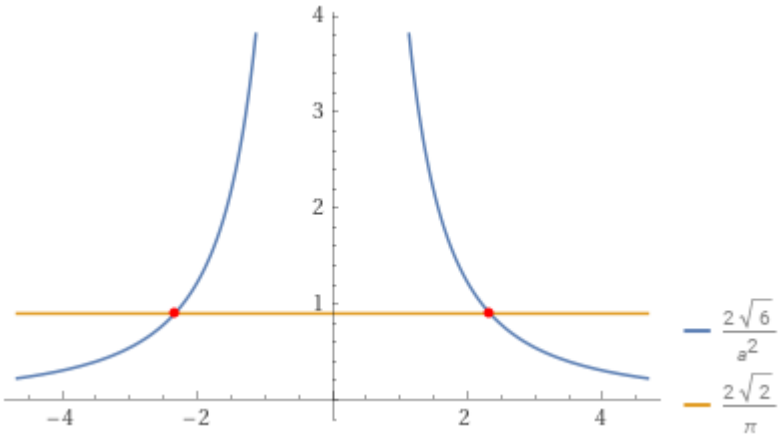
If we consider $(2\sqrt{2})/\pi$ equal to $(2\sqrt{6})/a^2$, we obtain:

$((2\sqrt{6})/a^2) = ((2\sqrt{2})/\pi)$ = DN Constant

Input

$$\frac{2\sqrt{6}}{a^2} = \frac{2\sqrt{2}}{\pi}$$

Plot **(figures that can be related to the open strings)**



Alternate form assuming a is real

$$a = \frac{\sqrt{3}\,\pi}{a}$$

Alternate form

$$a^2 = \sqrt{3}\,\pi \quad (\text{for } a \neq 0)$$

Alternate form assuming a is positive

$$a^2 = \sqrt{3}\,\pi \quad (\text{for } a \neq 0)$$

Solutions

$a = -\sqrt[4]{3}\ \sqrt{\pi}$

$a = \sqrt[4]{3}\ \sqrt{\pi}$

From

$$c_n = \frac{2\sqrt{6}}{a^2}\left[\int_0^{a/2} x\sin\left(\frac{n\pi}{a}x\right)dx - \int_{a/2}^{a} x\sin\left(\frac{n\pi}{a}x\right)dx + a\int_{a/2}^{a}\sin\left(\frac{n\pi}{a}x\right)dx\right]$$

$$I_1 - I_2 = \frac{a^2}{n\pi}\cos n\pi + \frac{2a^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) - \frac{a^2}{n\pi}\cos\left(\frac{n\pi}{2}\right)$$

$$I_3 = a\int_{a/2}^{a}\sin\left(\frac{n\pi}{a}x\right)dx = a\left[-\frac{a}{n\pi}\cos\left(\frac{n\pi}{a}x\right)\right]_{a/2}^{a}$$

$$I_3 = \left[-\frac{a^2}{n\pi}\cos n\pi + \frac{a^2}{n\pi}\cos(\frac{n\pi}{2})\right]$$

$$I = I_1 - I_2 + I_3 = \frac{2a^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

For $a^2 = (\sqrt{3}\ \pi)$

(((2√6)/(√3 π)))*((((((√3 π)/(nπ) cos (n π))) + ((2(√3 π))/(n^2 π^2) sin (1/2*n π)) - (((√3 π)/(nπ) cos (1/2*n π))))))

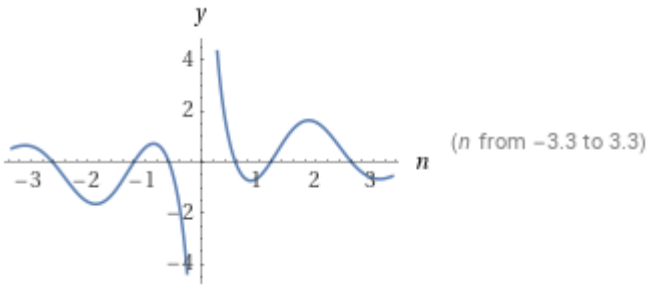
Input

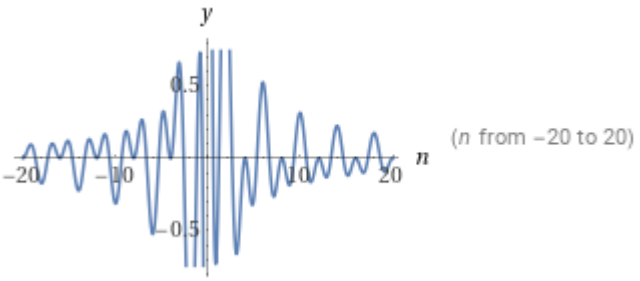
$$\frac{2\sqrt{6}}{\sqrt{3}\,\pi}\left(\frac{\sqrt{3}\,\pi}{n\,\pi}\cos(n\,\pi)+\frac{2\left(\sqrt{3}\,\pi\right)}{n^2\,\pi^2}\sin\left(\frac{1}{2}\,n\,\pi\right)-\frac{\sqrt{3}\,\pi}{n\,\pi}\cos\left(\frac{1}{2}\,n\,\pi\right)\right)$$

Exact result

$$\frac{2\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}$$

Plots





Alternate forms

$$-\frac{4\sqrt{6}\,\sin\left(\frac{\pi n}{4}\right)\left(\pi\,n\sin\left(\frac{3\pi n}{4}\right)-2\cos\left(\frac{\pi n}{4}\right)\right)}{\pi^2\,n^2}$$

$$-\frac{2\sqrt{6}\,\left(-2\sin\left(\frac{\pi n}{2}\right)+\pi\,n\cos\left(\frac{\pi n}{2}\right)-\pi\,n\cos(\pi n)\right)}{\pi^2\,n^2}$$

$$\frac{2\sqrt{6}\left(2\sin\left(\frac{\pi n}{2}\right)+\pi(-n)\cos\left(\frac{\pi n}{2}\right)+\pi n\cos(\pi n)\right)}{\pi^2 n^2}$$

Expanded trigonometric form

$$\frac{2\sqrt{6}\,(-1)\cos\left(\frac{1}{2}n\pi\right)}{\pi n}+\frac{2\sqrt{6}\,\cos(n\pi)}{\pi n}+\frac{4\sqrt{6}\,\sin\left(\frac{1}{2}n\pi\right)}{\pi^2 n^2}$$

Reduced trigonometric form

$$\frac{2\sqrt{2}\left(-\sqrt{3}\,n\pi\cos\left(\frac{1}{2}n\pi\right)+\sqrt{3}\,n\pi\cos(n\pi)+2\sqrt{3}\,\sin\left(\frac{1}{2}n\pi\right)\right)}{\left(n^2\pi\right)\pi}$$

Numerical roots

$n = \pm 4$

$n \approx \pm 2.61839251411086\ldots$

$n \approx \pm 1.19828673844203\ldots$

$n \approx \pm 0.582546134797964\ldots$

Property as a function

Parity

odd

Series expansion at n=0

$$\frac{2\sqrt{6}}{\pi n} - \frac{5\pi n}{\sqrt{6}} + \frac{19\pi^3 n^3}{40\sqrt{6}} - \frac{221\pi^5 n^5}{13440\sqrt{6}} + O(n^6)$$

(Laurent series)

Derivative

$$\frac{d}{dn}\left(\frac{\left(2\sqrt{6}\right)\left(\frac{(\sqrt{3}\ \pi)\cos(n\pi)}{n\pi} + \frac{(2(\sqrt{3}\ \pi))\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2} - \frac{(\sqrt{3}\ \pi)\cos\left(\frac{n\pi}{2}\right)}{n\pi}\right)}{\sqrt{3}\ \pi}\right) =$$

$$\frac{\sqrt{6}\left(\pi^2 n^2 \sin\left(\frac{\pi n}{2}\right) - 2\pi^2 n^2 \sin(\pi n) - 8\sin\left(\frac{\pi n}{2}\right) + 4\pi n\cos\left(\frac{\pi n}{2}\right) - 2\pi n\cos(\pi n)\right)}{\pi^2 n^3}$$

Indefinite integral

$$\int \frac{2\sqrt{2}\left(-\frac{\sqrt{3}\cos\left(\frac{n\pi}{2}\right)}{n} + \frac{\sqrt{3}\cos(n\pi)}{n} + \frac{2\sqrt{3}\sin\left(\frac{n\pi}{2}\right)}{n^2\pi}\right)}{\pi}\,dn =$$

$$\frac{2\sqrt{6}\left(\pi n\,\mathrm{Ci}(n\pi) - 2\sin\left(\frac{\pi n}{2}\right)\right)}{\pi^2 n} + \text{constant}$$

Ci(x) is the cosine integral

Limit

$$\lim_{n\to\pm\infty} \frac{2\sqrt{2}\left(-\frac{\sqrt{3}\cos\left(\frac{n\pi}{2}\right)}{n} + \frac{\sqrt{3}\cos(n\pi)}{n} + \frac{2\sqrt{3}\sin\left(\frac{n\pi}{2}\right)}{n^2\pi}\right)}{\pi} = 0$$

(((2√6)/(√3 π))) ((-(√3 π)/(n π))*(cos (n π)) + ((√3 π)/(n π))*(cos(1/2*n π)))

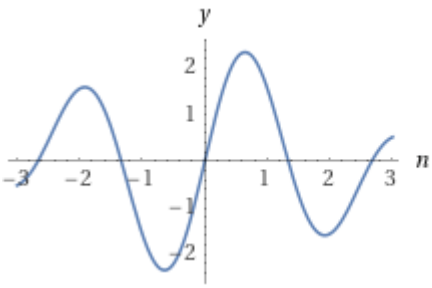
Input

$$\frac{2\sqrt{6}}{\sqrt{3}\ \pi}\left(-\frac{\sqrt{3}\ \pi}{n\pi}\cos(n\pi) + \frac{\sqrt{3}\ \pi}{n\pi}\cos\left(\frac{1}{2}n\pi\right)\right)$$

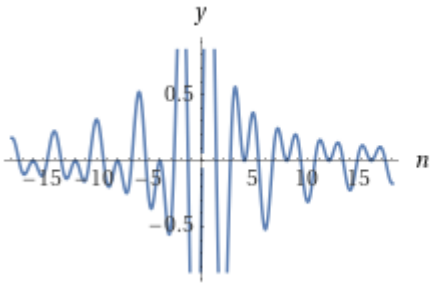
Exact result

$$\frac{2\sqrt{2}\left(\frac{\sqrt{3}\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\cos(\pi n)}{n}\right)}{\pi}$$

Plots (figures that can be related to the open strings)



(n from −3 to 3)



(n from −18 to 18)

Alternate forms

$$\frac{2\sqrt{6}\left(\cos\left(\frac{\pi n}{2}\right)-\cos(\pi n)\right)}{\pi n}$$

$$-\frac{2\sqrt{6}\left(\cos(\pi n)-\cos\left(\frac{\pi n}{2}\right)\right)}{\pi n}$$

$$\frac{4\sqrt{6}\,\sin^2\left(\frac{\pi n}{4}\right)\left(2\cos\left(\frac{\pi n}{2}\right)+1\right)}{\pi n}$$

Expanded trigonometric form

$$\frac{2\sqrt{6}\,\cos\left(\frac{1}{2}n\pi\right)}{\pi n}+\frac{2\sqrt{6}\,(-1)\cos(n\pi)}{\pi n}$$

Reduced trigonometric form

$$\frac{2\sqrt{2}\left(\sqrt{3}\,\cos\left(\frac{1}{2}n\pi\right)-\sqrt{3}\,\cos(n\pi)\right)}{n\pi}$$

Roots

$n=4m,\quad m\neq 0,\quad m\in\mathbb{Z}$

$n=\frac{4}{3}(3m-1),\quad m\in\mathbb{Z}$

$n=\frac{4}{3}(3m+1),\quad m\in\mathbb{Z}$

$\mathbb{Z}$ is the set of integers

Property as a function

Parity

odd

Series expansion at n=0

$$\frac{3}{2}\sqrt{\frac{3}{2}}\,\pi n-\frac{5}{32}\left(\sqrt{\frac{3}{2}}\,\pi^3\right)n^3+\frac{7\sqrt{\frac{3}{2}}\,\pi^5 n^5}{1280}+O\left(n^6\right)$$

(Taylor series)

Derivative

$$\frac{d}{dn}\left(\frac{\left(2\sqrt{6}\right)\left(-\frac{\left(\sqrt{3}\,\pi\right)\cos(n\pi)}{n\pi}+\frac{\left(\sqrt{3}\,\pi\right)\cos\left(\frac{n\pi}{2}\right)}{n\pi}\right)}{\sqrt{3}\,\pi}\right)=$$
$$\frac{\sqrt{6}\,\left(-\pi n\left(\sin\left(\frac{\pi n}{2}\right)-2\sin(\pi n)\right)-2\cos\left(\frac{\pi n}{2}\right)+2\cos(\pi n)\right)}{\pi n^2}$$

Indefinite integral

$$\int\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{n\pi}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(n\pi)}{n}\right)}{\pi}\,dn=\frac{2\sqrt{6}\,\left(\mathrm{Ci}\left(\frac{n\pi}{2}\right)-\mathrm{Ci}(n\pi)\right)}{\pi}+\text{constant}$$

Ci(x) is the cosine integral

Limit

$$\lim_{n\to\pm\infty}\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{n\pi}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(n\pi)}{n}\right)}{\pi}=0$$

From

$$\frac{2\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}$$

and

$$\frac{2\sqrt{2}\left(\frac{\sqrt{3}\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\cos(\pi n)}{n}\right)}{\pi}$$

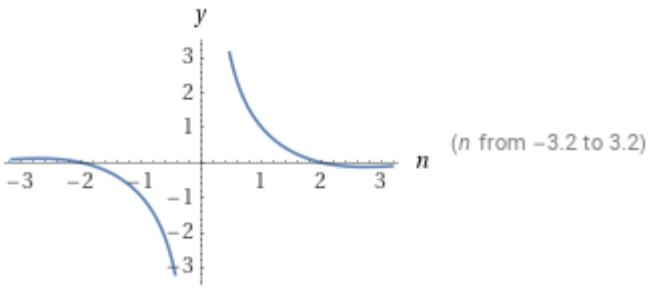
we obtain:

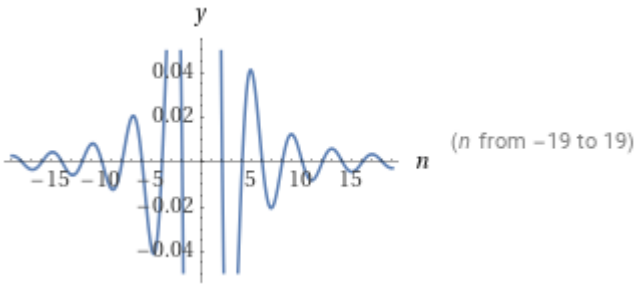
(((2 sqrt(2) ((2 sqrt(3) sin((π n)/2))/(π n^2) - (sqrt(3) cos((π n)/2))/n + (sqrt(3) cos(π n))/n))/π))+(((2 sqrt(2) ((sqrt(3) cos((π n)/2))/n - (sqrt(3) cos(π n))/n))/π))

Input

$$\frac{2\sqrt{2}\left(\frac{2\sqrt{3}\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\cos(\pi n)}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(\frac{\sqrt{3}\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\cos(\pi n)}{n}\right)}{\pi}$$

Plots **(figures that can be related to the open strings)**





Alternate forms

$$\frac{8\sqrt{6}\,\sin\left(\frac{\pi n}{4}\right)\cos\left(\frac{\pi n}{4}\right)}{\pi^2 n^2}$$

$$\frac{2i\sqrt{6}\,e^{-1/2\,i\pi n}}{\pi^2 n^2} - \frac{2i\sqrt{6}\,e^{(i\pi n)/2}}{\pi^2 n^2}$$

Reduced trigonometric form

$$\frac{4\sqrt{6}\,\sin\left(\frac{1}{2}n\pi\right)}{n^2\pi^2}$$

Roots

$$n = 2m, \quad m \neq 0, \quad m \in \mathbb{Z}$$

$\mathbb{Z}$ is the set of integers

Integer root

$$n = 4m + 2, \quad m \in \mathbb{Z}$$

Property as a function

Parity

odd

Series expansion at n=0

$$\frac{2\sqrt{6}}{\pi n} - \frac{\pi n}{2\sqrt{6}} + \frac{\pi^3 n^3}{160\sqrt{6}} - \frac{\pi^5 n^5}{26880\sqrt{6}} + O\left(n^6\right)$$

(Laurent series)

Derivative

$$\frac{d}{dn}\left(\frac{2\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}\right)=\frac{2\sqrt{6}\left(\pi n\cos\left(\frac{\pi n}{2}\right)-4\sin\left(\frac{\pi n}{2}\right)\right)}{\pi^2 n^3}$$

Indefinite integral

$$\int\left(\frac{2\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}\right)dn=\frac{2\sqrt{6}\left(\pi n\,\mathrm{Ci}\left(\frac{n\pi}{2}\right)-2\sin\left(\frac{\pi n}{2}\right)\right)}{\pi^2 n}+\text{constant}$$

Ci(x) is the cosine integral

Local minima

$$\min\left\{\frac{2\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}\right\}\approx -0.121413\ \text{at}\ n\approx 2.72141$$

$$\min\left\{\frac{2\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}\right\}\approx -0.0410479\ \text{at}\ n\approx -4.83611$$

Local maxima

$$\max\left\{\frac{2\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}\right\}\approx 0.121413 \text{ at } n\approx -2.72141$$

$$\max\left\{\frac{2\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)}{\pi}\right\}\approx 0.0410479 \text{ at } n\approx 4.83611$$

Limit

$$\lim_{n\to\pm\infty}\left(\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{n\pi}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(n\pi)}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(-\frac{\sqrt{3}\,\cos\left(\frac{n\pi}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(n\pi)}{n}+\frac{2\sqrt{3}\,\sin\left(\frac{n\pi}{2}\right)}{n^2\pi}\right)}{\pi}\right)=0$$

Definite integral

$$\int_{\frac{1}{2}}^{1}\left(\frac{2\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{n\pi}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(n\pi)}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(-\frac{\sqrt{3}\,\cos\left(\frac{n\pi}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(n\pi)}{n}+\frac{2\sqrt{3}\,\sin\left(\frac{n\pi}{2}\right)}{n^2\pi}\right)}{\pi}\right)dn\approx 0.858211\ldots$$

Alternative representations

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{2\sqrt{2}\left(\frac{\cosh\left(\frac{i n \pi}{2}\right)\sqrt{3}}{n}-\frac{\cosh(i n \pi)\sqrt{3}}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(-\frac{\cosh\left(\frac{i n \pi}{2}\right)\sqrt{3}}{n}+\frac{\cosh(i n \pi)\sqrt{3}}{n}+\frac{2\cos\left(\frac{\pi}{2}-\frac{n\pi}{2}\right)\sqrt{3}}{\pi n^2}\right)}{\pi}$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{2\sqrt{2}\left(\frac{\cosh\left(-\frac{1}{2}i n \pi\right)\sqrt{3}}{n}-\frac{\cosh(-i n \pi)\sqrt{3}}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(-\frac{\cosh\left(-\frac{1}{2}i n \pi\right)\sqrt{3}}{n}+\frac{\cosh(-i n \pi)\sqrt{3}}{n}+\frac{2\cos\left(\frac{\pi}{2}-\frac{n\pi}{2}\right)\sqrt{3}}{\pi n^2}\right)}{\pi}$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{2\sqrt{2}\left(\frac{\cosh\left(-\frac{1}{2}i n \pi\right)\sqrt{3}}{n}-\frac{\cosh(-i n \pi)\sqrt{3}}{n}\right)}{\pi}+\frac{2\sqrt{2}\left(-\frac{\cosh\left(-\frac{1}{2}i n \pi\right)\sqrt{3}}{n}+\frac{\cosh(-i n \pi)\sqrt{3}}{n}-\frac{2\cos\left(\frac{\pi}{2}+\frac{n\pi}{2}\right)\sqrt{3}}{\pi n^2}\right)}{\pi}$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{4\sqrt{6}\,\sum_{k=0}^{\infty}\frac{(-1)^k(-1+n)^{2k}\left(\frac{\pi}{2}\right)^{2k}}{(2k)!}}{n^2\pi^2}$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{4\sqrt{6}\,\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-1-2k} n^{1+2k}\pi^{1+2k}}{(1+2k)!}}{n^2\pi^2}$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{\sqrt{\frac{6}{\pi}}\,\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{16^s\left(n^2\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}{n}$$

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Integral representations

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{2\sqrt{6}}{n\pi}\int_0^1\cos\left(\frac{n\pi t}{2}\right)dt$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{2\sqrt{6}}{n\pi}\int_0^1\cos\left(\frac{n\pi t}{2}\right)dt \quad \text{for } \gamma>0$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{2\sqrt{6}}{n\pi}\int_0^1\cos\left(\frac{n\pi t}{2}\right)dt \quad \text{for } \left(0<\gamma<\frac{1}{2} \text{ and } n>0\right)$$

Half-argument formulas

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{4\sqrt{3-3\cos(n\pi)}}{n^2\pi^2}$$
$$\text{for } (0<\pi\,\mathrm{Re}(n)<2\pi \text{ or } (\mathrm{Re}(n)=0 \text{ and } \mathrm{Im}(n)\geq 0) \text{ or } (\mathrm{Re}(n)=2 \text{ and } \mathrm{Im}(n)\leq 0))$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=-\frac{4\,(-1)^{\lfloor \mathrm{Re}(n)/2\rfloor}\sqrt{3-3\cos(n\pi)}\left(-1+\left(1+(-1)^{\lfloor -\mathrm{Re}(n)/2\rfloor+\lfloor \mathrm{Re}(n)/2\rfloor}\right)\theta(-\pi\,\mathrm{Im}(n))\right)}{n^2\pi^2}$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{4\sqrt{3-3\cos(n\pi)}}{n^2\pi^2}\quad\text{for}$$

$$((\pi|\mathrm{Re}(n)|<\pi\text{ or }(\pi\,\mathrm{Re}(n)=-\pi\text{ and }\pi\,\mathrm{Im}(n)\geq 0)\text{ or }(\pi\,\mathrm{Re}(n)=\pi\text{ and }\pi\,\mathrm{Im}(n)\leq 0))$$
$$\text{and }(2\pi|\mathrm{Re}(n)|<\pi\text{ or }(2\pi\,\mathrm{Re}(n)=-\pi\text{ and }2\pi\,\mathrm{Im}(n)\geq 0)\text{ or}$$
$$(2\pi\,\mathrm{Re}(n)=\pi\text{ and }2\pi\,\mathrm{Im}(n)\leq 0))\text{ and }(0<\pi\,\mathrm{Re}(n)<2\pi\text{ or}$$
$$(\pi\,\mathrm{Re}(n)=0\text{ and }\pi\,\mathrm{Im}(n)\geq 0)\text{ or }(\pi\,\mathrm{Re}(n)=2\pi\text{ and }\pi\,\mathrm{Im}(n)\leq 0)))$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=$$
$$\frac{2\sqrt{2}\left(\frac{\sqrt{\frac{3}{2}}\,\sqrt{1+\cos(n\pi)}}{n}-\frac{\sqrt{\frac{3}{2}}\,\sqrt{1+\cos(2n\pi)}}{n}\right)}{\pi}+$$
$$\frac{1}{\pi}2\sqrt{2}\left(-\frac{\sqrt{\frac{3}{2}}\,\sqrt{1+\cos(n\pi)}}{n}+\frac{\sqrt{\frac{3}{2}}\,\sqrt{1+\cos(2n\pi)}}{n}+\frac{1}{n^2\pi}(-1)^{\lfloor \mathrm{Re}(n)/2\rfloor}\right.$$
$$\left.\sqrt{6}\,\sqrt{1-\cos(n\pi)}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(n)/2\rfloor+\lfloor \mathrm{Re}(n)/2\rfloor}\right)\theta(-\pi\,\mathrm{Im}(n))\right)\right)\quad\text{for}$$

$$((\pi|\mathrm{Re}(n)|<\pi\text{ or }(\pi\,\mathrm{Re}(n)=-\pi\text{ and }\pi\,\mathrm{Im}(n)\geq 0)\text{ or }(\pi\,\mathrm{Re}(n)=\pi\text{ and }\pi\,\mathrm{Im}(n)\leq 0))$$
$$\text{and }(2\pi|\mathrm{Re}(n)|<\pi\text{ or }(2\pi\,\mathrm{Re}(n)=-\pi\text{ and }2\pi\,\mathrm{Im}(n)\geq 0)$$
$$\text{or }(2\pi\,\mathrm{Re}(n)=\pi\text{ and }2\pi\,\mathrm{Im}(n)\leq 0)))$$

$\mathrm{Re}(z)$ is the real part of z

$\mathrm{Im}(z)$ is the imaginary part of z

$\lfloor x \rfloor$ is the floor function

$\theta(x)$ is the Heaviside step function

$|z|$ is the absolute value of z

Multiple-argument formulas

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{4\sqrt{6}\,\sin\left(\frac{n\pi}{2}\right)}{n^2\,\pi^2}$$

$$\frac{2\left(\sqrt{2}\left(\frac{2\sqrt{3}\,\sin\left(\frac{\pi n}{2}\right)}{\pi n^2}-\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}+\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}+\frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}\,\cos\left(\frac{\pi n}{2}\right)}{n}-\frac{\sqrt{3}\,\cos(\pi n)}{n}\right)\right)}{\pi}=\frac{4\sqrt{6}\,\left(1+2\cos\left(\frac{n\pi}{3}\right)\right)\sin\left(\frac{n\pi}{6}\right)}{n^2\,\pi^2}$$

From the alternate form

$$\frac{8\sqrt{6}\,\sin\left(\frac{\pi n}{4}\right)\cos\left(\frac{\pi n}{4}\right)}{\pi^2\,n^2}$$

(8 sqrt(6) cos((n π)/4) sin((n π)/4))/(n^2 π^2)

for n = π:

(8 sqrt(6) cos((π^2)/4) sin((π^2)/4))/(π^2 π^2)

Input

$$\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}$$

Exact result

$$\frac{8\sqrt{6}\,\sin\left(\frac{\pi^2}{4}\right)\cos\left(\frac{\pi^2}{4}\right)}{\pi^4}$$

Decimal approximation

−0.0981080438261807053234525603542009121308487988434449356709706 72

…

-0.0981080438….

Alternate forms

$$\frac{4\sqrt{6}\,\sin\left(\frac{\pi^2}{2}\right)}{\pi^4}$$

$$\frac{2i\sqrt{6}\,e^{-(i\pi^2)/2}}{\pi^4}-\frac{2i\sqrt{6}\,e^{(i\pi^2)/2}}{\pi^4}$$

Alternative representations

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}=\frac{8\cosh\left(\frac{i\pi^2}{4}\right)\cos\left(\frac{\pi}{2}-\frac{\pi^2}{4}\right)\sqrt{6}}{(\pi^2)^2}$$

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}=-\frac{8\cosh\left(-\frac{i\pi^2}{4}\right)\cos\left(\frac{\pi}{2}+\frac{\pi^2}{4}\right)\sqrt{6}}{(\pi^2)^2}$$

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}=\frac{8\cosh\left(-\frac{i\pi^2}{4}\right)\cos\left(\frac{\pi}{2}-\frac{\pi^2}{4}\right)\sqrt{6}}{(\pi^2)^2}$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2} = \frac{8\sqrt{6}\,\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,4^{-1-2k_1-2k_2}\,\pi^{2+4k_1+4k_2}}{(2k_1)!\,(1+2k_2)!}}{\pi^4}$$

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2} = \frac{8\sqrt{6}\,\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,4^{-2k_1-2k_2}\,(-2+\pi)^{2k_2}\,\pi^{4k_1+2k_2}}{(2k_1)!\,(2k_2)!}}{\pi^4}$$

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2} = -\frac{8\sqrt{6}\,\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,4^{-2-2k_1-2k_2}\,(-2+\pi)^{1+2k_2}\,\pi^{3+4k_1+2k_2}}{(1+2k_1)!\,(1+2k_2)!}}{\pi^4}$$

$n!$ is the factorial function

Integral representations

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2} = -\frac{i\sqrt{6}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^4/(64\,s)+s}}{\sqrt{s}}\,ds\right)\int_0^1\cos\left(\frac{\pi^2 t}{4}\right)dt}{\pi^{5/2}} \quad \text{for } \gamma > 0$$

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2} = -\frac{\sqrt{\frac{3}{2}}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^4/(64\,s)+s}}{s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^4/(64\,s)+s}}{\sqrt{s}}\,ds}{2\pi^3} \quad \text{for } \gamma > 0$$

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2} = \frac{i\sqrt{\frac{3}{2}}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^4/(64\,s)+s}}{s^{3/2}}\,ds\right)\int_{\frac{\pi}{2}}^{\frac{\pi^2}{4}}\sin(t)\,dt}{\pi^{5/2}} \quad \text{for } \gamma > 0$$

Multiple-argument formulas

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2} = \frac{16\sqrt{6}\,\cos\left(\frac{\pi^2}{8}\right)\left(-1+2\cos^2\left(\frac{\pi^2}{8}\right)\right)\sin\left(\frac{\pi^2}{8}\right)}{\pi^4}$$

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2} = \frac{16\sqrt{6}\,\cos\left(\frac{\pi^2}{8}\right)\sin\left(\frac{\pi^2}{8}\right)\left(1-2\sin^2\left(\frac{\pi^2}{8}\right)\right)}{\pi^4}$$

$$\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2} = \frac{8\sqrt{6}\;T_{\frac{1}{4}}\left(\cos(\pi^2)\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^4}$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

From which, after some calculations:

(-1/((8 sqrt(6) cos((π^2)/4) sin((π^2)/4))/(π^2 π^2)))^3+9^3-55-4

Input

$$\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3 + 9^3 - 55 - 4$$

Exact result

$$670 - \frac{\pi^{12}\csc^3\left(\frac{\pi^2}{4}\right)\sec^3\left(\frac{\pi^2}{4}\right)}{3072\sqrt{6}}$$

csc(x) is the cosecant function

sec(x) is the secant function

Decimal approximation

1728.976080793170834757302646972380071740696747252617948586692499
9…

1728.97608079317….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2.

Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$670 - \frac{\pi^{12} \csc^3\left(\frac{\pi^2}{2}\right)}{384\sqrt{6}}$$

$$\frac{12\,349\,440 - \sqrt{6}\,\pi^{12} \csc^3\left(\frac{\pi^2}{4}\right) \sec^3\left(\frac{\pi^2}{4}\right)}{18\,432}$$

$$-\frac{\left(\sqrt{6}\,\pi^{12} - 1\,157\,760 \sin\left(\frac{\pi^2}{2}\right) + 385\,920 \sin\left(\frac{3\pi^2}{2}\right)\right) \csc^3\left(\frac{\pi^2}{4}\right) \sec^3\left(\frac{\pi^2}{4}\right)}{18\,432}$$

Alternative representations

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3 + 9^3 - 55 - 4 = -59 + 9^3 + \left(-\frac{1}{\frac{8\cosh\left(\frac{i\pi^2}{4}\right)\cos\left(\frac{\pi}{2} - \frac{\pi^2}{4}\right)\sqrt{6}}{(\pi^2)^2}}\right)^3$$

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3 + 9^3 - 55 - 4 = -59 + 9^3 + \left(\frac{-1}{-\frac{8\cosh\left(-\frac{i\pi^2}{4}\right)\cos\left(\frac{\pi}{2} + \frac{\pi^2}{4}\right)\sqrt{6}}{(\pi^2)^2}}\right)^3$$

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=-59+9^3+\left(-\frac{1}{\frac{8\cosh\left(-\frac{i\,\pi^2}{4}\right)\cos\left(\frac{\pi}{2}-\frac{\pi^2}{4}\right)\sqrt{6}}{(\pi^2)^2}}\right)^3$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=670-\frac{i\,\pi^{12}\sec^3\left(\frac{\pi^2}{4}\right)\left(\sum_{k=1}^{\infty}q^{-1+2k}\right)^3}{384\sqrt{6}}$$

for $q=e^{(i\,\pi^2)/4}$

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=$$

$$\frac{1}{288}\left(192\,960+i\sqrt{6}\,\pi^{12}\left(\sum_{k=1}^{\infty}q^{-1+2k}\right)^3\left(\sum_{k=1}^{\infty}(-1)^k\,q^{-1+2k}\right)^3\right)\quad\text{for } q=e^{(i\,\pi^2)/4}$$

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=670-\frac{\pi^{18}\sec^3\left(\frac{\pi^2}{4}\right)\left(\sum_{k=-\infty}^{\infty}\frac{16(-1)^k}{-16k^2\,\pi^2+\pi^4}\right)^3}{196\,608\sqrt{6}}$$

Integral representations

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=$$

$$670+\frac{8\,\pi^6}{\left(\int_0^1\cos\left(\frac{\pi^2\,t}{4}\right)dt\right)^3\left(-4+\pi^2\int_0^1\sin\left(\frac{\pi^2\,t}{4}\right)dt\right)^3\sqrt{6}^{\,3}}$$

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=$$

$$670+\frac{512\,i^3\,\pi^9}{\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^4/(64\,s)+s}}{s^{3/2}}\,ds\right)^3\left(-4+\pi^2\int_0^1\sin\left(\frac{\pi^2\,t}{4}\right)dt\right)^3\sqrt{6}^{\,3}\sqrt{\pi}^{\,3}}\quad\text{for}\ \gamma>0$$

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=$$

$$\frac{\pi^6+5360\left(\int_0^1\cos\left(\frac{\pi^2\,t}{4}\right)dt\right)^3\left(\int_{\frac{\pi}{2}}^{\frac{\pi^2}{4}}\sin(t)\,dt\right)^3\sqrt{6}^{\,3}}{8\left(\int_0^1\cos\left(\frac{\pi^2\,t}{4}\right)dt\right)^3\left(\int_{\frac{\pi}{2}}^{\frac{\pi^2}{4}}\sin(t)\,dt\right)^3\sqrt{6}^{\,3}}$$

Multiple-argument formulas

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=670-\frac{\pi^{12}\csc^3\left(\frac{\pi^2}{2}\right)}{384\sqrt{6}}$$

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=670-\frac{\pi^{12}\csc^3\left(\frac{\pi^2}{8}\right)\sec^9\left(\frac{\pi^2}{8}\right)}{24576\sqrt{6}\left(2-\sec^2\left(\frac{\pi^2}{8}\right)\right)^3}$$

$$\left(-\frac{1}{\frac{8\left(\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4=670-\frac{\pi^{12}\csc^3\left(\frac{\pi^2}{8}\right)\sec^3\left(\frac{\pi^2}{8}\right)}{24576\sqrt{6}\;T_{\frac{1}{4}}\left(\cos\left(\pi^2\right)\right)^3}$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

(1/27(((-1/((8 sqrt(6) cos((π^2)/4) sin((π^2)/4))/(π^2 π^2)))^3+9^3-55-4)-1))^2+1/π^2

Input

$$\left(\frac{1}{27}\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}$$

Exact result

$$\frac{1}{\pi^2}+\frac{1}{729}\left(669-\frac{\pi^{12}\csc^3\left(\frac{\pi^2}{4}\right)\sec^3\left(\frac{\pi^2}{4}\right)}{3072\sqrt{6}}\right)^2$$

csc(x) is the cosecant function

sec(x) is the secant function

Decimal approximation

4095.98792721015380588889673500163314331202333767608366771642203 62

…

4095.98792721…. ≈ 4096

The number 4096 = 64^2 = 8^4, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson.
This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{1}{\pi^2}+\frac{\left(\sqrt{6}\,\pi^{12}\csc^3\left(\frac{\pi^2}{2}\right)-1\,541\,376\right)^2}{3\,869\,835\,264}$$

$$\frac{1}{41\,278\,242\,816\,\pi^2}\left(41\,278\,242\,816+25\,342\,293\,049\,344\,\pi^2+\pi^{26}\csc^6\left(\frac{\pi^2}{4}\right)\sec^6\left(\frac{\pi^2}{4}\right)-4\,110\,336\sqrt{6}\,\pi^{14}\csc^3\left(\frac{\pi^2}{4}\right)\sec^3\left(\frac{\pi^2}{4}\right)\right)$$

$$\frac{1}{\pi^2}+\frac{1}{729}\left(669-\frac{i\,\pi^{12}}{48\sqrt{6}\left(e^{-(i\,\pi^2)/4}-e^{(i\,\pi^2)/4}\right)^3\left(e^{-(i\,\pi^2)/4}+e^{(i\,\pi^2)/4}\right)^3}\right)^2$$

Expanded form

$$\frac{49\,729}{81}+\frac{1}{\pi^2}+\frac{\pi^{24}\csc^6\left(\frac{\pi^2}{4}\right)\sec^6\left(\frac{\pi^2}{4}\right)}{41\,278\,242\,816}-\frac{223\,\pi^{12}\csc^3\left(\frac{\pi^2}{4}\right)\sec^3\left(\frac{\pi^2}{4}\right)}{373\,248\sqrt{6}}$$

Alternative representations

$$\left(\frac{1}{27}\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$\frac{1}{\pi^2}+\left(\frac{1}{27}\left(-60+9^3+\left(-\frac{1}{\frac{8\cosh\left(\frac{i\,\pi^2}{4}\right)\cos\left(\frac{\pi}{2}-\frac{\pi^2}{4}\right)\sqrt{6}}{(\pi^2)^2}}\right)^3\right)\right)^2$$

$$\left(\frac{1}{27}\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$\frac{1}{\pi^2}+\left(\frac{1}{27}\left(-60+9^3+\left(\frac{-1}{-\frac{8\cosh\left(-\frac{i\,\pi^2}{4}\right)\cos\left(\frac{\pi}{2}+\frac{\pi^2}{4}\right)\sqrt{6}}{(\pi^2)^2}}\right)^3\right)\right)^2$$

$$\left(\frac{1}{27}\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$\frac{1}{\pi^2}+\left(\frac{1}{27}\left(-60+9^3+\left(-\frac{1}{\frac{8\cosh\left(-\frac{i\,\pi^2}{4}\right)\cos\left(\frac{\pi}{2}-\frac{\pi^2}{4}\right)\sqrt{6}}{(\pi^2)^2}}\right)^3\right)\right)^2$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations

$$\left(\frac{1}{27}\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$-\frac{1}{644\,972\,544\,\pi^2}\left(-644\,972\,544-395\,973\,328\,896\,\pi^2+513\,792\,i\,\sqrt{6}\,\pi^{14}\right.$$
$$\left.\sec^3\left(\frac{\pi^2}{4}\right)\left(\sum_{k=1}^{\infty}q^{-1+2k}\right)^3+\pi^{26}\sec^6\left(\frac{\pi^2}{4}\right)\left(\sum_{k=1}^{\infty}q^{-1+2k}\right)^6\right)\quad\text{for } q=e^{(i\,\pi^2)/4}$$

$$\left(\frac{1}{27}\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$\frac{1}{\pi^2}+\frac{\left(12\,331\,008+\sqrt{6}\,\pi^6\left(8+16\,\pi^4\sum_{k=1}^{\infty}\frac{(-1)^k}{-16\,k^2\,\pi^2+\pi^4}\right)^3\left(\sum_{k=1}^{\infty}(-1)^k\,q^{-1+2k}\right)^3\right)^2}{247\,669\,456\,896}$$
$$\text{for } q=e^{(i\,\pi^2)/4}$$

$$\left(\frac{1}{27}\left(\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$-\frac{1}{10077696\,\pi^2}\left(-10077696-6187083264\,\pi^2-\right.$$
$$64224\,i\,\sqrt{6}\,\pi^{14}\left(\sum_{k=1}^{\infty}q^{-1+2k}\right)^3\left(\sum_{k=1}^{\infty}(-1)^k\,q^{-1+2k}\right)^3+$$
$$\left.\pi^{26}\left(\sum_{k=1}^{\infty}q^{-1+2k}\right)^6\left(\sum_{k=1}^{\infty}(-1)^k\,q^{-1+2k}\right)^6\right)\quad\text{for } q=e^{(i\,\pi^2)/4}$$

Integral representations

$$\left(\frac{1}{27}\left(\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$\frac{49729}{81}+\frac{1}{\pi^2}+\frac{64\,\pi^{12}}{729\left(\int_0^1\cos\left(\frac{\pi^2\,t}{4}\right)dt\right)^6\left(-4+\pi^2\int_0^1\sin\left(\frac{\pi^2\,t}{4}\right)dt\right)^6\sqrt{6}^6}+$$
$$\frac{3568\,\pi^6}{243\left(\int_0^1\cos\left(\frac{\pi^2\,t}{4}\right)dt\right)^3\left(-4+\pi^2\int_0^1\sin\left(\frac{\pi^2\,t}{4}\right)dt\right)^3\sqrt{6}^3}$$

$$\left(\frac{1}{27}\left(\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$\frac{49729}{81}+\frac{1}{\pi^2}+\frac{64\,i^6\,\pi^{18}}{729\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^4/(64\,s)+s}}{s^{3/2}}\,ds\right)^6\left(1-\frac{\pi^2}{4}\int_0^1\sin\left(\frac{\pi^2\,t}{4}\right)dt\right)^6\sqrt{6}^6\sqrt{\pi}^6}-$$
$$\frac{3568\,i^3\,\pi^9}{243\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^4/(64\,s)+s}}{s^{3/2}}\,ds\right)^3\left(1-\frac{\pi^2}{4}\int_0^1\sin\left(\frac{\pi^2\,t}{4}\right)dt\right)^3\sqrt{6}^3\sqrt{\pi}^3}\quad\text{for }\gamma>0$$

$$\left(\frac{1}{27}\left(\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$\left(\pi^{14}+10704\,\pi^8\left(\int_0^1\cos\left(\frac{\pi^2\,t}{4}\right)dt\right)^3\left(\int_{\frac{\pi}{2}}^{\frac{\pi^2}{4}}\sin(t)\,dt\right)^3\sqrt{6}^3+\right.$$
$$46656\left(\int_0^1\cos\left(\frac{\pi^2\,t}{4}\right)dt\right)^6\left(\int_{\frac{\pi}{2}}^{\frac{\pi^2}{4}}\sin(t)\,dt\right)^6\sqrt{6}^6+$$
$$\left.28643904\,\pi^2\left(\int_0^1\cos\left(\frac{\pi^2\,t}{4}\right)dt\right)^6\left(\int_{\frac{\pi}{2}}^{\frac{\pi^2}{4}}\sin(t)\,dt\right)^6\sqrt{6}^6\right)\Big/$$
$$\left(46656\,\pi^2\left(\int_0^1\cos\left(\frac{\pi^2\,t}{4}\right)dt\right)^6\left(\int_{\frac{\pi}{2}}^{\frac{\pi^2}{4}}\sin(t)\,dt\right)^6\sqrt{6}^6\right)$$

Multiple-argument formulas

$$\left(\frac{1}{27}\left(\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$\frac{1}{\pi^2}+\frac{1}{729}\left(669-\frac{\pi^{12}\csc^3\left(\frac{\pi^2}{8}\right)\sec^9\left(\frac{\pi^2}{8}\right)}{24576\sqrt{6}\left(2-\sec^2\left(\frac{\pi^2}{8}\right)\right)^3}\right)^2$$

$$\left(\frac{1}{27}\left(\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\,\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$
$$\frac{1}{\pi^2}+\frac{1}{729}\left(669-\frac{\pi^{12}\csc^3\left(\frac{\pi^2}{8}\right)\sec^3\left(\frac{\pi^2}{8}\right)}{24576\sqrt{6}\;T_{\frac{1}{4}}\left(\cos\left(\pi^2\right)\right)^3}\right)^2$$

$$\left(\frac{1}{27}\left(\left(\left(-\frac{1}{\frac{8\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\right)-1\right)\right)^2+\frac{1}{\pi^2}=$$

$$\frac{1}{\pi^2}+\frac{1}{729}\left(669-\frac{\pi^{12}\csc^9\left(\frac{\pi^2}{12}\right)}{3072\sqrt{6}\;T_{\frac{1}{4}}\left(\cos\left(\pi^2\right)\right)^3\left(-4+3\csc^2\left(\frac{\pi^2}{12}\right)\right)^3}\right)^2$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

((-1/((8 sqrt(6) cos((π^2)/4) sin((π^2)/4))/(π^2 π^2)))^3+9^3-55-4)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{\left(-\frac{1}{\frac{8\sqrt{6}\cos\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi^2}{4}\right)}{\pi^2\,\pi^2}}\right)^3+9^3-55-4\;+C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}}$$

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+\sqrt[15]{670-\frac{\pi^{12}\csc^3\left(\frac{\pi^2}{4}\right)\sec^3\left(\frac{\pi^2}{4}\right)}{3072\sqrt{6}}}$$

$\csc(x)$ is the cosecant function

$\sec(x)$ is the secant function

Decimal approximation

1.644936508070484409547252841510422403037172873951554987182780427 3…

1.644936508…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\sqrt[15]{670-\frac{\pi^{12}\csc^3\left(\frac{\pi^2}{2}\right)}{384\sqrt{6}}}$$

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\frac{\sqrt[15]{12\,349\,440-\sqrt{6}\,\pi^{12}\csc^3\left(\frac{\pi^2}{4}\right)\sec^3\left(\frac{\pi^2}{4}\right)}}{2^{11/15}\times 3^{2/15}}$$

$$\frac{1}{6}C_{\text{MRB}}^{\,-1/(4\pi)}\left(6C_{\text{MRB}}^{\,1+\pi}+2^{4/15}\times 3^{13/15}\sqrt[15]{12\,349\,440-\sqrt{6}\,\pi^{12}\csc^3\left(\frac{\pi^2}{4}\right)\sec^3\left(\frac{\pi^2}{4}\right)}\sqrt[4\pi]{C_{\text{MRB}}}\right)$$

From the previous exact result

$$\frac{4\sqrt{6}\,\sin\left(\frac{1}{2}n\pi\right)}{n^2\pi^2}$$

multiplied by the Grablovitz expression

$$(u+v)t^2+[(u+v)]t-\{[(u+v)^2-uv]-(u+v)t\}-\frac{1}{2}u^2v^2=0$$

we obtain:

(((4 sqrt(6) sin(1/2 n π))/(n^2 π^2)))(((u+v) t^2+[(u+v)]t-{[(u+v)^2-uv]-(u+v)t}-1/2 u^2 v^2))

Input

$$\frac{4\sqrt{6}\,\sin\left(\frac{1}{2}n\pi\right)}{n^2\pi^2}\left((u+v)t^2+(u+v)t-\left(\left((u+v)^2-uv\right)-(u+v)t\right)-\frac{1}{2}u^2v^2\right)$$

Exact result

$$\frac{4\sqrt{6}\,\sin\left(\frac{\pi n}{2}\right)\left(t^2(u+v)+2t(u+v)-\frac{u^2v^2}{2}-(u+v)^2+uv\right)}{\pi^2n^2}$$

Solutions

$$u=\left(\left(-\frac{4\sqrt{6}\,t^2\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}-\frac{8\sqrt{6}\,t\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}+\frac{4\sqrt{6}\,v\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}\right)\pm\sqrt{\left(\left(\frac{4\sqrt{6}\,t^2\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}+\frac{8\sqrt{6}\,t\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}-\frac{4\sqrt{6}\,v\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}\right)^2-4\left(-\frac{2\sqrt{6}\,v^2\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}-\frac{4\sqrt{6}\,\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}\right)\left(\frac{4\sqrt{6}\,t^2v\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}+\frac{8\sqrt{6}\,tv\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}-\frac{4\sqrt{6}\,v^2\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}\right)\right)}\Bigg/\left(2\left(-\frac{2\sqrt{6}\,v^2\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}-\frac{4\sqrt{6}\,\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}\right)\right)\quad\left(\frac{\left(v^2+2\right)\sin\left(\frac{\pi n}{2}\right)}{n}\neq 0\right)$$

Alternate forms assuming n, t, u, and v are positive

$$\frac{2\sqrt{6}\,\sin\left(\frac{\pi n}{2}\right)\left(2t^2(u+v)+4t(u+v)-u^2\left(v^2+2\right)-2uv-2v^2\right)}{\pi^2n^2}$$

$$\frac{4\sqrt{6}\,t^2\sin\left(\frac{\pi n}{2}\right)(u+v)}{\pi^2n^2}+\frac{8\sqrt{6}\,t\sin\left(\frac{\pi n}{2}\right)(u+v)}{\pi^2n^2}-\frac{2\sqrt{6}\,u^2v^2\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}-\frac{4\sqrt{6}\,\sin\left(\frac{\pi n}{2}\right)(u+v)^2}{\pi^2n^2}+\frac{4\sqrt{6}\,uv\sin\left(\frac{\pi n}{2}\right)}{\pi^2n^2}$$

Alternate forms

$$-\frac{2\sqrt{6}\,\sin\left(\frac{\pi n}{2}\right)\left(-2t^2u-2t^2v-4tu-4tv+u^2v^2+2u^2+2uv+2v^2\right)}{\pi^2n^2}$$

$$\frac{2i\sqrt{6}\,\left(e^{-1/2i\pi n}-e^{(i\pi n)/2}\right)\left(t^2(u+v)+2t(u+v)-\frac{u^2v^2}{2}-(u+v)^2+uv\right)}{\pi^2n^2}$$

Reduced trigonometric form

$$\frac{2\sqrt{6}\,\left(4tu+2t^2u-2u^2+4tv+2t^2v-2uv-2v^2-u^2v^2\right)\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}$$

Expanded sine form

$$\frac{8\sqrt{6}\,tu\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}+\frac{4\sqrt{6}\,t^2u\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}-\frac{4\sqrt{6}\,u^2\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}+\frac{8\sqrt{6}\,tv\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}+\frac{4\sqrt{6}\,t^2v\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}-\frac{4\sqrt{6}\,uv\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}-\frac{4\sqrt{6}\,v^2\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}-\frac{2\sqrt{6}\,u^2v^2\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}$$

Derivative

$$\frac{\partial}{\partial u}\left(\frac{\left(4\sqrt{6}\,\sin\left(\frac{n\pi}{2}\right)\right)\left((u+v)t^2+(u+v)t-\left(\left((u+v)^2-uv\right)-(u+v)t\right)-\frac{u^2v^2}{2}\right)}{n^2\pi^2}\right)=$$

$$\frac{4\sqrt{6}\,\sin\left(\frac{\pi n}{2}\right)\left(t^2+2t-u\left(v^2+2\right)-v\right)}{\pi^2n^2}$$

Indefinite integral

$$\int \frac{4\sqrt{6}\left(u\,v-\frac{u^2\,v^2}{2}+2\,t\,(u+v)+t^2\,(u+v)-(u+v)^2\right)\sin\left(\frac{n\pi}{2}\right)}{n^2\,\pi^2}\,du=$$
$$\frac{4\sqrt{6}\,\sin\left(\frac{\pi\,n}{2}\right)\left(\frac{t^2\,u^2}{2}+t^2\,u\,v+t\,u^2+2\,t\,u\,v-\frac{1}{6}\,u^3\,v^2+\frac{u^2\,v}{2}-\frac{1}{3}\,(u+v)^3\right)}{\pi^2\,n^2}+\text{constant}$$

Alternative representations

$$\frac{\left((u+v)\,t^2+(u+v)\,t-\left(\left((u+v)^2-u\,v\right)-(u+v)\,t\right)-\frac{u^2\,v^2}{2}\right)\left(4\sqrt{6}\,\sin\left(\frac{n\pi}{2}\right)\right)}{n^2\,\pi^2}=$$
$$\frac{4\left(u\,v+2\,t\,(u+v)+(u+v)\,t^2-\frac{u^2\,v^2}{2}-(u+v)^2\right)\sqrt{6}}{\csc\left(\frac{n\pi}{2}\right)\left(n^2\,\pi^2\right)}$$

$$\frac{\left((u+v)\,t^2+(u+v)\,t-\left(\left((u+v)^2-u\,v\right)-(u+v)\,t\right)-\frac{u^2\,v^2}{2}\right)\left(4\sqrt{6}\,\sin\left(\frac{n\pi}{2}\right)\right)}{n^2\,\pi^2}=$$
$$\frac{4\cos\left(\frac{\pi}{2}-\frac{n\pi}{2}\right)\left(u\,v+2\,t\,(u+v)+(u+v)\,t^2-\frac{u^2\,v^2}{2}-(u+v)^2\right)\sqrt{6}}{n^2\,\pi^2}$$

$$\frac{\left((u+v)\,t^2+(u+v)\,t-\left(\left((u+v)^2-u\,v\right)-(u+v)\,t\right)-\frac{u^2\,v^2}{2}\right)\left(4\sqrt{6}\,\sin\left(\frac{n\pi}{2}\right)\right)}{n^2\,\pi^2}=$$
$$-\frac{4\cos\left(\frac{\pi}{2}+\frac{n\pi}{2}\right)\left(u\,v+2\,t\,(u+v)+(u+v)\,t^2-\frac{u^2\,v^2}{2}-(u+v)^2\right)\sqrt{6}}{n^2\,\pi^2}$$

$\csc(x)$ is the cosecant function

From the alternate form

$$-\frac{2\sqrt{6}\,\sin\left(\frac{\pi n}{2}\right)\left(-2t^2u-2t^2v-4tu-4tv+u^2v^2+2u^2+2uv+2v^2\right)}{\pi^2 n^2}$$

-(2 sqrt(6) (-4 t u - 2 t^2 u + 2 u^2 - 4 t v - 2 t^2 v + 2 u v + 2 v^2 + u^2 v^2) sin((n π)/2))/(n^2 π^2)

and

$$u=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}=\sqrt[3]{-\frac{3}{2}+\sqrt{\frac{3^2}{4}+\frac{2^3}{27}}}=0.457427107\ldots$$

$$v=\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}=\sqrt[3]{-\frac{3}{2}-\sqrt{\frac{3^2}{4}+\frac{2^3}{27}}}=-1.457427107\ldots$$

we obtain:

-(2√(6)(-4 t (0.4574271)-2 t^2 (0.4574271)+2 (0.4574271)^2-4t(-1.4574271)-2 t^2 (-1.4574271)+2(0.4574271)(-1.4574271)+2(-1.4574271)^2+(0.4574271)^2 (-1.4574271)^2) sin((nπ)/2))/(n^2 π^2)

Input interpretation

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4t\times0.4574271+2t^2\times(-0.4574271)+2\times0.4574271^2-\right.$$
$$4t\times(-1.4574271)-2t^2\times(-1.4574271)+2\times0.4574271\times(-1.4574271)+$$
$$\left.2(-1.4574271)^2+0.4574271^2\,(-1.4574271)^2\right)\sin\left(\frac{n\pi}{2}\right)$$

Result

$$-\frac{2\sqrt{6}\,\left(2t^2+4t+3.77778\right)\sin\left(\frac{\pi n}{2}\right)}{\pi^2 n^2}$$

Solutions

$$t = -0.503656\, n^2 \csc\left(\frac{\pi n}{2}\right)\left(\frac{1.98548 \sin\left(\frac{\pi n}{2}\right)}{n^2} \pm 1.87193 \sqrt{-\frac{\sin^2\left(\frac{\pi n}{2}\right)}{n^4}}\right)$$

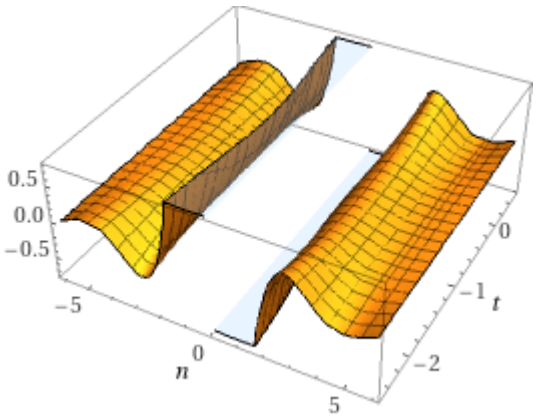
$$\left(\frac{\sin\left(\frac{\pi n}{2}\right)}{n} \neq 0\right)$$

csc(x) is the cosecant function

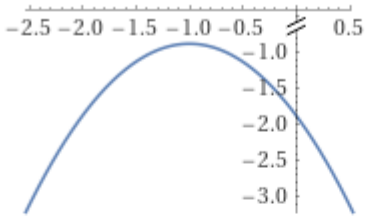
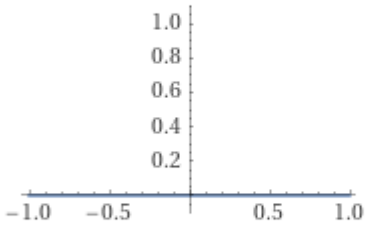
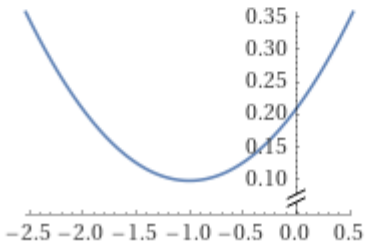
Values

n	
1	$-\frac{2\sqrt{6}\left(2t^2+4t+3.77778\right)}{\pi^2}$
2	0
3	$\frac{2\sqrt{\frac{2}{3}}\left(2t^2+4t+3.77778\right)}{3\pi^2}$

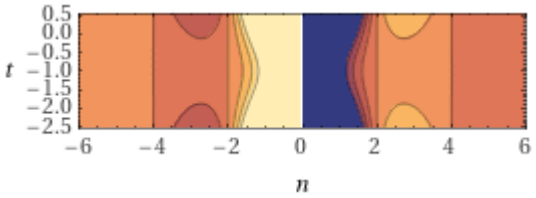
3D plot (figures that can be related to the D-branes/Instantons)



Plots (figures that can be related to the open strings)

n	
1	 (t from −2.5 to 0.5)
2	 (t from −1 to 1)
3	 (t from −2.5 to 0.5)

Contour plot



Alternate forms

$$-\frac{0.992741\left(t^2+2\,t+1.88889\right)\sin\left(\frac{\pi n}{2}\right)}{n^2}$$

$$\frac{((-0.992741\,t-1.98548)\,t-1.87518)\sin\left(\frac{\pi n}{2}\right)}{n^2}$$

$$-\frac{4\sqrt{6}\left(t^2+2t+1.88889\right)\sin\left(\frac{\pi n}{2}\right)}{\pi^2 n^2}$$

Alternate form assuming n and t are positive

$$\frac{\left(-0.992741\,t^2-1.98548\,t-1.87518\right)\sin\left(\frac{\pi n}{2}\right)}{n^2}$$

Expanded trigonometric form

$$-\frac{18.5073\sin\left(\frac{1}{2}n\pi\right)}{n^2\pi^2}-\frac{19.5959\,t\sin\left(\frac{1}{2}n\pi\right)}{n^2\pi^2}-\frac{0.992741\,t^2\sin\left(\frac{1}{2}n\pi\right)}{n^2}$$

Reduced trigonometric form

$$-\frac{0.992741\left(t^2\sin\left(\frac{\pi n}{2}\right)+2t\sin\left(\frac{\pi n}{2}\right)+1.88889\sin\left(\frac{\pi n}{2}\right)\right)}{n^2}$$

Property as a real function

Domain

$\mathbb{R}$ (all real numbers)

$\mathbb{R}$ is the set of real numbers

Derivative

$$\frac{\partial}{\partial t}\left(-\frac{2\sqrt{6}\left(2t^2+4t+3.77778\right)\sin\left(\frac{\pi n}{2}\right)}{\pi^2 n^2}\right)=-\frac{1.98548\,(t+1)\sin\left(\frac{\pi n}{2}\right)}{n^2}$$

Indefinite integral

$$\int -\frac{2\sqrt{6}\left(3.77778+4\,t+2\,t^2\right)\sin\left(\frac{n\pi}{2}\right)}{n^2\,\pi^2}\,dt=$$
$$\frac{\left(-0.330914\,t^3-0.992741\,t^2-1.87518\,t\right)\sin(1.5708\,n)}{n^2}+\text{constant}$$

Alternative representations

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times0.457427^2-\right.$$
$$-1.45743\times4\,t--1.45743\times2\,t^2+2\times0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$-\frac{1}{\csc\left(\frac{n\pi}{2}\right)\left(n^2\,\pi^2\right)}2\left(-1.33333+4\,t+2\,(-1.45743)^2+\right.$$
$$\left.2\times0.457427^2+(-1.45743)^2\,0.457427^2+2\,t^2\right)\sqrt{6}$$

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times0.457427^2-\right.$$
$$-1.45743\times4\,t--1.45743\times2\,t^2+2\times0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$\frac{1}{n^2\,\pi^2}2\cos\left(\frac{\pi}{2}+\frac{n\,\pi}{2}\right)\left(-1.33333+4\,t+2\,(-1.45743)^2+\right.$$
$$\left.2\times0.457427^2+(-1.45743)^2\,0.457427^2+2\,t^2\right)\sqrt{6}$$

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times0.457427^2-\right.$$
$$-1.45743\times4\,t--1.45743\times2\,t^2+2\times0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$-\frac{1}{n^2\,\pi^2}2\cos\left(\frac{\pi}{2}-\frac{n\,\pi}{2}\right)\left(-1.33333+4\,t+2\,(-1.45743)^2+\right.$$
$$\left.2\times0.457427^2+(-1.45743)^2\,0.457427^2+2\,t^2\right)\sqrt{6}$$

Series representations

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times 0.457427^2-\right.$$
$$-1.45743\times 4\,t--1.45743\times 2\,t^2+2\times 0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$\frac{0.631998\left(1.88889+2\,t+t^2\right)\sum_{k=1}^{\infty}\frac{(-1)^k\,k\,\sin(k\,n\,\pi)}{-\frac{1}{4}+k^2}}{n^2}\quad\text{for}$$
$$-1<n<1$$

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times 0.457427^2-\right.$$
$$-1.45743\times 4\,t--1.45743\times 2\,t^2+2\times 0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$-\frac{1.98548\left(1.88889+2\,t+t^2\right)\sum_{k=0}^{\infty}(-1)^k\,J_{1+2\,k}\left(\frac{n\,\pi}{2}\right)}{n^2}$$

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times 0.457427^2-\right.$$
$$-1.45743\times 4\,t--1.45743\times 2\,t^2+2\times 0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$-\frac{0.992741\left(1.88889+2\,t+t^2\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2\,k}\,n^{1+2\,k}\,\pi^{1+2\,k}}{(1+2\,k)!}}{n^2}$$

$J_n(z)$ is the Bessel function of the first kind

$n!$ is the factorial function

Integral representations

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times 0.457427^2-\right.$$
$$-1.45743\times 4\,t--1.45743\times 2\,t^2+2\times 0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$-\frac{1.55939\left(1.88889+2\,t+t^2\right)}{n}\int_0^1\cos\left(\frac{n\,\pi\,\tau}{2}\right)d\tau$$

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times0.457427^2-\right.$$
$$-1.45743\times4\,t--1.45743\times2\,t^2+2\times0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$\frac{(0.219948\,i)\left(1.88889+2\,t+t^2\right)}{n}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-(n^2\,\pi^2)/(16\,s)+s}}{s^{3/2}}\,ds\quad\text{for}$$
$\gamma>0$

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times0.457427^2-\right.$$
$$-1.45743\times4\,t--1.45743\times2\,t^2+2\times0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$\frac{(0.280047\,i)\left(1.88889+2\,t+t^2\right)}{n^2}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{n^{1-2\,s}\left(\frac{\pi}{4}\right)^{1-2\,s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds$$
for $(0<\gamma<1$ and $n>0)$

$\Gamma(x)$ is the gamma function

Half-argument formulas

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times0.457427^2-\right.$$
$$-1.45743\times4\,t--1.45743\times2\,t^2+2\times0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$-\frac{0.701974\left(1.88889+2\,t+t^2\right)\sqrt{1-\cos(n\,\pi)}}{n^2}$$
for $(0<\pi\,\mathrm{Re}(n)<2\,\pi$ or $(\mathrm{Re}(n)=0$ and $\mathrm{Im}(n)\geq0)$ or $(\mathrm{Re}(n)=2$ and $\mathrm{Im}(n)\leq0))$

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times0.457427^2-\right.$$
$$-1.45743\times4\,t--1.45743\times2\,t^2+2\times0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$\frac{1}{n^2}0.701974\,(-1)^{\lfloor\mathrm{Re}(n)/2\rfloor}\left(1.88889+2\,t+t^2\right)\sqrt{1-\cos(n\,\pi)}$$
$$\left(-1+\left(1+(-1)^{\lfloor-\mathrm{Re}(n)/2\rfloor+\lfloor\mathrm{Re}(n)/2\rfloor}\right)\theta(-\pi\,\mathrm{Im}(n))\right)$$

$\mathrm{Re}(z)$ is the real part of z

$\mathrm{Im}(z)$ is the imaginary part of z

$\lfloor x \rfloor$ is the floor function

$\theta(x)$ is the Heaviside step function

Multiple-argument formulas

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times 0.457427^2-\right.$$
$$-1.45743\times 4\,t--1.45743\times 2\,t^2+2\times 0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$-\frac{0.992741\left(1.88889+2\,t+t^2\right)\sin\left(\frac{n\,\pi}{2}\right)}{n^2}$$

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times 0.457427^2-\right.$$
$$-1.45743\times 4\,t--1.45743\times 2\,t^2+2\times 0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$\frac{3.97096\left(1.88889+2\,t+t^2\right)\left(-0.75\sin\left(\frac{n\,\pi}{6}\right)+\sin^3\left(\frac{n\,\pi}{6}\right)\right)}{n^2}$$

$$-\frac{1}{n^2\,\pi^2}2\sqrt{6}\,\left(-4\,t\,0.457427-2\,t^2\,0.457427+2\times 0.457427^2-\right.$$
$$-1.45743\times 4\,t--1.45743\times 2\,t^2+2\times 0.457427\,(-1.45743)+$$
$$\left.2\,(-1.45743)^2+0.457427^2\,(-1.45743)^2\right)\sin\left(\frac{n\,\pi}{2}\right)=$$
$$-\frac{2.97822\left(1.88889+2\,t+t^2\right)\left(\cos^2\left(\frac{n\,\pi}{6}\right)\sin\left(\frac{n\,\pi}{6}\right)-0.333333\sin^3\left(\frac{n\,\pi}{6}\right)\right)}{n^2}$$

From the Reduced trigonometric form

$$-\frac{0.992741\left(t^2\sin\left(\frac{\pi\,n}{2}\right)+2\,t\sin\left(\frac{\pi\,n}{2}\right)+1.88889\sin\left(\frac{\pi\,n}{2}\right)\right)}{n^2}$$

-(0.992741 (1.88889 sin((n π)/2) + 2 t sin((n π)/2) + t^2 sin((n π)/2)))/n^2

for n = t = -φ , we obtain:

-(0.992741 (1.88889 sin((-φ π)/2) + 2 (-φ) sin((-φ π)/2) + (-φ)^2 sin((-φ π)/2)))/(-φ)^2

Input interpretation

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}$$

ϕ is the golden ratio

Result

0.272097613939615252395768704046690520054846817616598396985444083 6

…

0.272097613939….

Alternative representations

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=-\frac{0.992741\left(\frac{1.88889}{\csc\left(-\frac{\phi\pi}{2}\right)}-\frac{2\phi}{\csc\left(-\frac{\phi\pi}{2}\right)}+\frac{(-\phi)^2}{\csc\left(-\frac{\phi\pi}{2}\right)}\right)}{(-\phi)^2}$$

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=-\frac{0.992741\left(1.88889\cos\left(\frac{\pi}{2}+\frac{\phi\pi}{2}\right)-2\phi\cos\left(\frac{\pi}{2}+\frac{\phi\pi}{2}\right)+\cos\left(\frac{\pi}{2}+\frac{\phi\pi}{2}\right)(-\phi)^2\right)}{(-\phi)^2}$$

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$-\frac{0.992741\left(-1.88889\cos\left(\frac{\pi}{2}-\frac{\phi\pi}{2}\right)+2\phi\cos\left(\frac{\pi}{2}-\frac{\phi\pi}{2}\right)-\cos\left(\frac{\pi}{2}-\frac{\phi\pi}{2}\right)(-\phi)^2\right)}{(-\phi)^2}$$

csc(x) is the cosecant function

Series representations

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$\frac{\left(-3.75036+3.97096\,\phi-1.98548\,\phi^2\right)\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{\phi\pi}{2}\right)}{\phi^2}$$

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$\frac{\left(-1.87518+1.98548\,\phi-0.992741\,\phi^2\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(-(1+\phi)\pi)^{2k}}{(2k)!}}{\phi^2}$$

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$\frac{\left(-1.87518+1.98548\,\phi-0.992741\,\phi^2\right)\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-1-2k}(-\phi\pi)^{1+2k}}{(1+2k)!}}{\phi^2}$$

$J_n(z)$ is the Bessel function of the first kind

$n!$ is the factorial function

Integral representations

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$\frac{\left(0.937589-0.992741\,\phi+0.496371\,\phi^2\right)\pi}{\phi}\int_0^1\cos\left(-\frac{1}{2}\phi\pi t\right)dt$$

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$\frac{\left(0.234397-0.248185\,\phi+0.124093\,\phi^2\right)\sqrt{\pi}}{\phi\,i}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-(\phi^2\pi^2)/(16\,s)+s}}{s^{3/2}}\,ds\quad\text{for }\gamma>0$$

i is the imaginary unit

Half-argument formula

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$-\frac{1}{\phi^2}0.992741\left(1.88889\,(-1)^{\lfloor\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\sqrt{\frac{1}{2}(1-\cos(-\phi\pi))}\left(1-\left(1+(-1)^{\lfloor-\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor+\lfloor\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(-\phi\pi))\right)-\right.$$
$$2\,(-1)^{\lfloor\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\,\phi\sqrt{\frac{1}{2}(1-\cos(-\phi\pi))}\left(1-\left(1+(-1)^{\lfloor-\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor+\lfloor\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(-\phi\pi))\right)+$$
$$(-1)^{\lfloor\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\,\phi^2\sqrt{\frac{1}{2}(1-\cos(-\phi\pi))}$$
$$\left.\left(1-\left(1+(-1)^{\lfloor-\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor+\lfloor\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(-\phi\pi))\right)\right)$$

$\mathrm{Re}(z)$ is the real part of z

$\lfloor x\rfloor$ is the floor function

$\mathrm{Im}(z)$ is the imaginary part of z

$\theta(x)$ is the Heaviside step function

Multiple-argument formulas

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$\frac{\left(-3.75036+3.97096\,\phi-1.98548\,\phi^2\right)\cos\left(-\frac{\phi\pi}{4}\right)\sin\left(-\frac{\phi\pi}{4}\right)}{\phi^2}$$

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$\frac{1}{\phi^2}\sin\left(-\frac{\phi\pi}{6}\right)\left(-5.62554+5.95645\,\phi-2.97822\,\phi^2+\left(7.50071-7.94193\,\phi+3.97096\,\phi^2\right)\sin^2\left(-\frac{\phi\pi}{6}\right)\right)$$

$$-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}=$$
$$\frac{\left(-1.87518+1.98548\,\phi-0.992741\,\phi^2\right)U_{-\frac{3}{2}}(\cos(\phi\pi))\sin(\phi\pi)}{\phi^2}$$

$U_n(x)$ is the Chebyshev polynomial of the second kind

From which:

1+1/(1+(2(-(0.992741 (1.88889 sin((-φ π)/2) + 2 (-φ) sin((-φ π)/2) + (-φ)^2 sin((-φ π)/2)))/(-φ)^2)))

Input interpretation

$$1+\frac{1}{1+2\left(-\frac{0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)}{(-\phi)^2}\right)}$$

ϕ is the golden ratio

Result

1.647586511048464848670761844159867230982362145276521336538494841 0…

1.647586511….≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternative representations

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$
$$1+\frac{1}{1-\frac{1.98548\left(\frac{1.88889}{\csc\left(-\frac{\phi\pi}{2}\right)}-\frac{2\phi}{\csc\left(-\frac{\phi\pi}{2}\right)}+\frac{(-\phi)^2}{\csc\left(-\frac{\phi\pi}{2}\right)}\right)}{(-\phi)^2}}$$

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$
$$1+\frac{1}{1-\frac{1.98548\left(1.88889\cos\left(\frac{\pi}{2}+\frac{\phi\pi}{2}\right)-2\phi\cos\left(\frac{\pi}{2}+\frac{\phi\pi}{2}\right)+\cos\left(\frac{\pi}{2}+\frac{\phi\pi}{2}\right)(-\phi)^2\right)}{(-\phi)^2}}$$

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$
$$1+\frac{1}{1-\frac{1.98548\left(-1.88889\cos\left(\frac{\pi}{2}-\frac{\phi\pi}{2}\right)+2\phi\cos\left(\frac{\pi}{2}-\frac{\phi\pi}{2}\right)-\cos\left(\frac{\pi}{2}-\frac{\phi\pi}{2}\right)(-\phi)^2\right)}{(-\phi)^2}}$$

$\csc(x)$ is the cosecant function

Series representations

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$
$$\left(-0.503656\,\phi^2+1.88889\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{\phi\pi}{2}\right)-\right.$$
$$\left.2\phi\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{\phi\pi}{2}\right)+\phi^2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{\phi\pi}{2}\right)\right)\Big/$$
$$\left(-0.251828\,\phi^2+1.88889\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{\phi\pi}{2}\right)-\right.$$
$$\left.2\phi\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{\phi\pi}{2}\right)+\phi^2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{\phi\pi}{2}\right)\right)$$

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$
$$\left(-1.00731\,\phi^2+1.88889\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(-(1+\phi)\pi)^{2k}}{(2k)!}-\right.$$
$$\left.2\phi\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(-(1+\phi)\pi)^{2k}}{(2k)!}+\phi^2\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(-(1+\phi)\pi)^{2k}}{(2k)!}\right)\Big/$$
$$\left(-0.503656\,\phi^2+1.88889\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(-(1+\phi)\pi)^{2k}}{(2k)!}-\right.$$
$$\left.2\phi\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(-(1+\phi)\pi)^{2k}}{(2k)!}+\phi^2\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(-(1+\phi)\pi)^{2k}}{(2k)!}\right)$$

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$

$$\left(-0.503656\,\phi^2+1.88889\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{1}{2}\right)T_{1+2k}(\phi\pi)-\right.$$
$$\left.2\phi\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{1}{2}\right)T_{1+2k}(\phi\pi)+\phi^2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{1}{2}\right)T_{1+2k}(\phi\pi)\right)\Big/$$
$$\left(-0.251828\,\phi^2+1.88889\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{1}{2}\right)T_{1+2k}(\phi\pi)-\right.$$
$$\left.2\phi\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{1}{2}\right)T_{1+2k}(\phi\pi)+\phi^2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(-\frac{1}{2}\right)T_{1+2k}(\phi\pi)\right)$$

$J_n(z)$ is the Bessel function of the first kind

$n!$ is the factorial function

$T_n(x)$ is the Chebyshev polynomial of the first kind

Integral representation

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$

$$\left(8.0585\,\phi\,i+1.88889\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\left(\phi^2\pi^2\right)/(16s)+s}}{s^{3/2}}\,ds-\right.$$
$$\left.2\phi\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\left(\phi^2\pi^2\right)/(16s)+s}}{s^{3/2}}\,ds+\phi^2\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\left(\phi^2\pi^2\right)/(16s)+s}}{s^{3/2}}\,ds\right)\Big/$$
$$\left(4.02925\,\phi\,i+1.88889\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\left(\phi^2\pi^2\right)/(16s)+s}}{s^{3/2}}\,ds-2\phi\sqrt{\pi}\right.$$
$$\left.\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\left(\phi^2\pi^2\right)/(16s)+s}}{s^{3/2}}\,ds+\phi^2\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\left(\phi^2\pi^2\right)/(16s)+s}}{s^{3/2}}\,ds\right)\quad\text{for}\ \gamma>0$$

i is the imaginary unit

Half-argument formula

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$

$$1+1\Bigg/\Bigg(1-\frac{1}{\phi^2}1.98548\Bigg(1.88889\,(-1)^{\lfloor \mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\sqrt{\frac{1}{2}(1-\cos(-\phi\pi))}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(-\phi\pi))\right)-2\,(-1)^{\lfloor \mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\phi\sqrt{\frac{1}{2}(1-\cos(-\phi\pi))}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(-\phi\pi))\right)+(-1)^{\lfloor \mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\phi^2\sqrt{\frac{1}{2}(1-\cos(-\phi\pi))}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(-\phi\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(-\phi\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(-\phi\pi))\right)\Bigg)\Bigg)$$

$\mathrm{Re}(z)$ is the real part of z

$\lfloor x \rfloor$ is the floor function

$\mathrm{Im}(z)$ is the imaginary part of z

$\theta(x)$ is the Heaviside step function

Multiple-argument formulas

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$

$$1-\frac{0.251828\,\phi^2}{-0.251828\,\phi^2+\left(1.88889-2\,\phi+\phi^2\right)\cos\left(-\frac{\phi\pi}{4}\right)\sin\left(-\frac{\phi\pi}{4}\right)}$$

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$

$$1-\frac{0.503656\,\phi^2}{-0.503656\,\phi^2+\left(1.88889-2\,\phi+\phi^2\right)U_{-\frac{3}{2}}(\cos(\phi\pi))\sin(\phi\pi)}$$

$$1+\frac{1}{1+\frac{2\left(-\left(0.992741\left(1.88889\sin\left(-\frac{\phi\pi}{2}\right)+2(-\phi)\sin\left(-\frac{\phi\pi}{2}\right)+(-\phi)^2\sin\left(-\frac{\phi\pi}{2}\right)\right)\right)\right)}{(-\phi)^2}}=$$

$$1+\left(0.125914\,\phi^2\right)\Big/\left(0.125914\,\phi^2+\left(-1.41667+1.5\,\phi-0.75\,\phi^2\right)\sin\left(-\frac{\phi\pi}{6}\right)+\left(1.88889-2\,\phi+\phi^2\right)\sin^3\left(-\frac{\phi\pi}{6}\right)\right)$$

$U_n(x)$ is the Chebyshev polynomial of the second kind

From

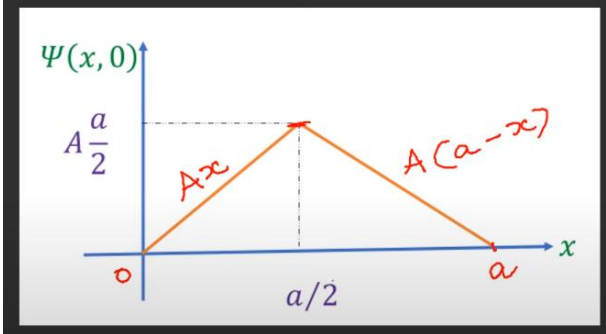
GS 2.7a Griffiths 3rd ed., quantum mechanics Problem 2.7 Part 1, related to normalization constant -
https://www.youtube.com/watch?v=qGp0a7Cqf24&list=FL2S91m-z1oo2gA0jDQ6oXHA&index=2&t=2s

We have that:

Problem 2.7 A particle in the infinite square well has the initial wave function

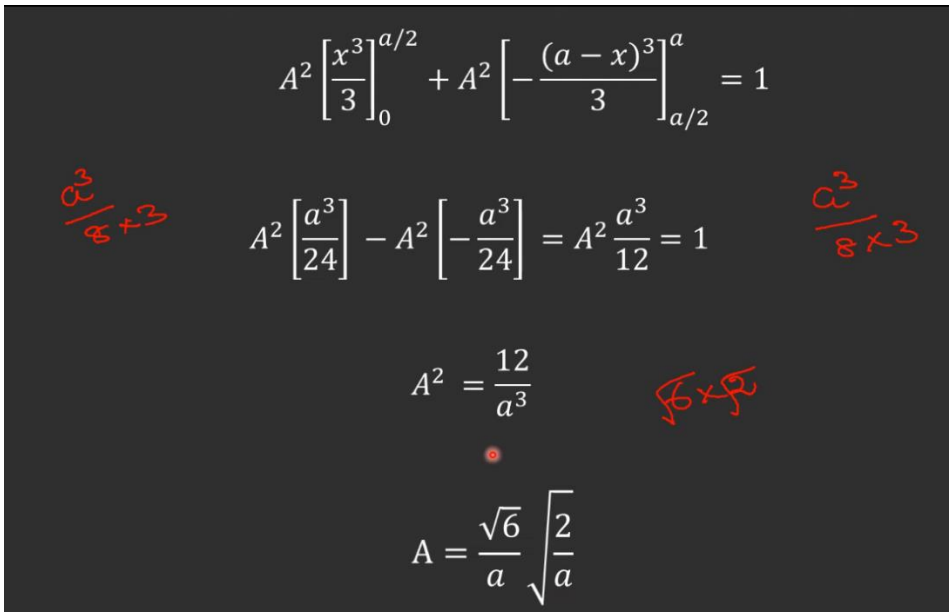
$$\Psi(x,0)=\begin{cases}Ax, & 0\le x\le a/2,\\ A\,(a-x), & a/2\le x\le a.\end{cases}$$

(a) Sketch $\Psi(x,0)$, and determine the constant A.



$$A^2\int_0^{a/2}x^2dx+A^2\int_{a/2}^{a}(a-x)^2dx=1$$

$$A^2\left[\frac{x^3}{3}\right]_0^{a/2}+A^2\left[-\frac{(a-x)^3}{3}\right]_{a/2}^{a}=1$$


$A^2\left[\frac{x^3}{3}\right]_0^{a/2} + A^2\left[-\frac{(a-x)^3}{3}\right]_{a/2}^{a} = 1$
$A^2\left[\frac{a^3}{24}\right] - A^2\left[-\frac{a^3}{24}\right] = A^2\frac{a^3}{12} = 1$
$A^2 = \frac{12}{a^3}$
$A = \frac{\sqrt{6}}{a}\sqrt{\frac{2}{a}}$

From

$$A = \frac{\sqrt{6}}{a}\sqrt{\frac{2}{a}}$$

placing: (sqrt6 / a)*(sqrt(2/a)) = (2√2)/π

we obtain:

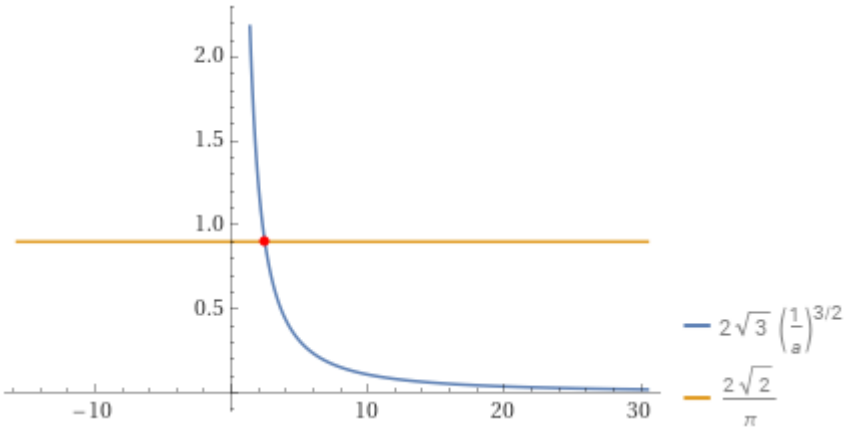
Input

$$\frac{\sqrt{6}}{a}\sqrt{\frac{2}{a}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$2\sqrt{3}\left(\frac{1}{a}\right)^{3/2} = \frac{2\sqrt{2}}{\pi}$$

Plot (figure that can be related to an open string)



Alternate form assuming a is real

$$\sqrt{3}\,\pi\left(\frac{1}{a}\right)^{3/2} = \sqrt{2}$$

Alternate form assuming a>0

$$\frac{2\sqrt{3}}{a^{3/2}} = \frac{2\sqrt{2}}{\pi}$$

Alternate form assuming a is positive

$$2a^3 = 3\pi^2$$

Solution

$$a = \sqrt[3]{\frac{3}{2}}\;\pi^{2/3}$$

Thus

$$\frac{\sqrt{6}}{a}\sqrt{\frac{2}{a}}$$

for

$$a = \sqrt[3]{\frac{3}{2}}\,\pi^{2/3}$$

we obtain:

(sqrt6 / ((3/2)^(1/3) π^(2/3)))*(sqrt(2/((3/2)^(1/3) π^(2/3))))

Input

$$\frac{\sqrt{6}}{\sqrt[3]{\frac{3}{2}}\,\pi^{2/3}}\sqrt{\frac{2}{\sqrt[3]{\frac{3}{2}}\,\pi^{2/3}}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157…≈ 0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant) (We note that, with regard the inflation, during a period of almost exponential expansion $\dot{H} < 0$ so that $\epsilon > 0$. Indeed, the value 0.9003163161571… = $(2\sqrt{2})/\pi > 0$ and also the n_s = spectral index

= 0.90-0.97 is near to the DN Constant value. Also the squared sound speed of the gravitino's longitudinal polarization mode could pass through 0.9003163161571 in the early universe, in the so-called quasi-de Sitter phase of inflation)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{\frac{2}{\sqrt[3]{\frac{3}{2}}\,\pi^{2/3}}}\,\sqrt{6}}{\sqrt[3]{\frac{3}{2}}\,\pi^{2/3}} = \frac{\sqrt[3]{\frac{2}{3}}\,\sqrt{z_0}^{\,2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(6-z_0)^{k_1}\left(\frac{2\sqrt[3]{\frac{2}{3}}}{\pi^{2/3}}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}}{\pi^{2/3}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{\sqrt{\frac{2}{\sqrt[3]{\frac{3}{2}}\,\pi^{2/3}}}\,\sqrt{6}}{\sqrt[3]{\frac{3}{2}}\,\pi^{2/3}} = \frac{1}{\pi^{2/3}}\sqrt[3]{\frac{2}{3}}\,\exp\left(i\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{2\sqrt[3]{\frac{2}{3}}}{\pi^{2/3}}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(6-x)^{k_1}\left(\frac{2\sqrt[3]{\frac{2}{3}}}{\pi^{2/3}}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\sqrt{\frac{2}{\sqrt[3]{\frac{3}{2}}\,\pi^{2/3}}}\,\sqrt{6}}{\sqrt[3]{\frac{3}{2}}\,\pi^{2/3}} = \frac{1}{\pi^{2/3}}\sqrt[3]{\frac{2}{3}}\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(6-z_0)/(2\pi)\rfloor+1/2\left\lfloor \arg\left(\frac{2\sqrt[3]{\frac{2}{3}}}{\pi^{2/3}}-z_0\right)\Big/(2\pi)\right\rfloor}$$

$$z_0^{1+1/2\lfloor \arg(6-z_0)/(2\pi)\rfloor+1/2\left\lfloor \arg\left(\frac{2\sqrt[3]{\frac{2}{3}}}{\pi^{2/3}}-z_0\right)\Big/(2\pi)\right\rfloor}$$

$$\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(6-z_0)^{k_1}\left(\frac{2\sqrt[3]{\frac{2}{3}}}{\pi^{2/3}}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}$$

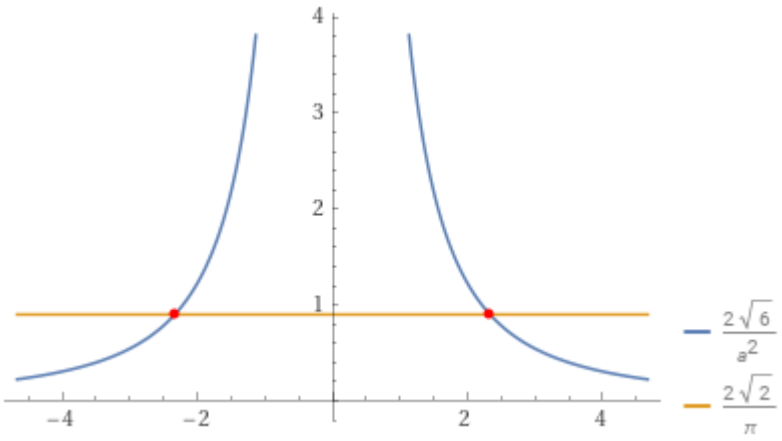
We have also:

(2√6)/(a^2)= (2√2)/π

Input

$$\frac{2\sqrt{6}}{a^2} = \frac{2\sqrt{2}}{\pi}$$

Plot **(figures that can be related to the open strings)**



Alternate form assuming a is real

$$a = \frac{\sqrt{3}\ \pi}{a}$$

Alternate form

$$a^2 = \sqrt{3}\ \pi \quad (\text{for } a \neq 0)$$

Alternate form assuming a is positive

$$a^2 = \sqrt{3}\ \pi \quad (\text{for } a \neq 0)$$

Solutions

$$a = -\sqrt[4]{3}\ \sqrt{\pi}$$

$$a = \sqrt[4]{3}\ \sqrt{\pi}$$

From

GS 2.37 Griffiths 3rd ed., Problem 2.37 Part 1, expectation value of time dependent wave function -
https://www.youtube.com/watch?v=K0gODNMd6bQ&list=FL2S91m-z1oo2gA0jDQ6oXHA&index=1&t=503s

We have that:

$$|A|^2 \frac{a}{2} \int_{-\infty}^{\infty} \left| \frac{3}{4}\psi_1 - \frac{1}{4}\psi_3 \right|^2 dx = |A|^2 \frac{a}{2} \int_{-\infty}^{\infty} \left[\frac{9}{16}|\psi_1|^2 + \frac{1}{16}|\psi_3|^2 - \cdots \right] dx = 1$$

$$\int \psi_1^* \psi_3 \, dx = 0 \qquad \langle \psi_1 | \psi_3 \rangle = 0 \qquad \int \psi_3^* \psi \, dx = 0$$

$$\int |\psi_1|^2 dx = 1 \qquad \int |\psi_3|^2 dx = 1$$

$$|A|^2 \frac{a}{2} \int_{-\infty}^{\infty} \left[\frac{9}{16}|\psi_1|^2 + \frac{1}{16}|\psi_3|^2 \right] dx = 1$$

$$|A|^2 \frac{a}{2} \left[\frac{9}{16} + \frac{1}{16} \right] = |A|^2 \frac{a}{2} \left[\frac{10}{16} \right] = |A|^2 \frac{5a}{16} = 1$$

$$|A|^2 = \frac{16}{5a} \longrightarrow A = \frac{4}{\sqrt{5a}}$$

From

$A = \frac{4}{\sqrt{5a}}$ for $a = \frac{2\sqrt{2}}{\pi}$

we obtain:

$$\frac{4}{\sqrt{5a}} = \frac{2\sqrt{2}}{\pi}$$

$4/\sqrt{5a}=(2\sqrt{2})/\pi$

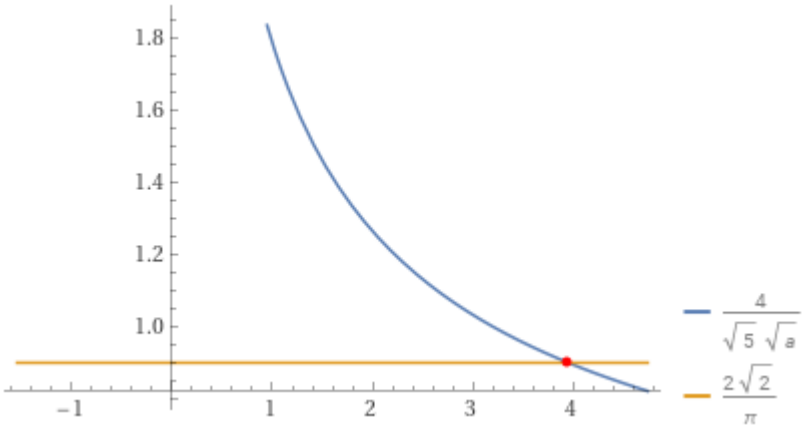
Input

$$\frac{4}{\sqrt{5\,a}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$\frac{4}{\sqrt{5}\ \sqrt{a}} = \frac{2\sqrt{2}}{\pi}$$

Plot **(figure that can be related to an open string)**



Alternate form assuming a is real

$$\frac{2\sqrt{5}\ \pi}{\sqrt{a}} = 5\sqrt{2}$$

Alternate form assuming a is positive

$$5a = 2\pi^2$$

Solution

$$a = \frac{2\pi^2}{5}$$

Thence, we obtain:

$$A = \frac{4}{\sqrt{5a}}$$

$$\frac{4}{\sqrt{5 \cdot \frac{2\pi^2}{5}}}$$

4/√(5·(2π^2)/5)

Input

$$\frac{4}{\sqrt{5\left(\frac{1}{5}\left(2\pi^2\right)\right)}}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157…≈ 0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant) (We note that, with regard the inflation, during a period of almost exponential expansion $\dot{H} < 0$ so that $\epsilon > 0$. Indeed, the value 0.9003163161571… = $(2\sqrt{2})/\pi > 0$ and also the n_s = spectral index = 0.90-0.97 is near to the DN Constant value. Also the squared sound speed of the gravitino's longitudinal polarization mode could pass through 0.9003163161571 in the early universe, in the so-called quasi-de Sitter phase of inflation)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{4}{\sqrt{\frac{5}{5}\left(2\pi^2\right)}} = \frac{4}{\sqrt{-1+2\pi^2}\sum_{k=0}^{\infty}\left(-1+2\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{4}{\sqrt{\frac{5}{5}\left(2\pi^2\right)}} = \frac{4}{\sqrt{-1+2\pi^2}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+2\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{4}{\sqrt{\frac{5}{5}\left(2\pi^2\right)}} = \frac{4}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(2\pi^2-z_0\right)^k z_0^{-k}}{k!}} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$\binom{n}{m}$ is the binomial coefficient

We have analyze the following expression:

$$c_n = \frac{2\sqrt{6}}{a^2}\left[\int_0^{a/2} x\sin\left(\frac{n\pi}{a}x\right)dx - \int_{a/2}^{a} x\sin\left(\frac{n\pi}{a}x\right)dx + a\int_{a/2}^{a}\sin\left(\frac{n\pi}{a}x\right)dx\right]$$

If in this expression concerning the solution of the Schrodinger equation for a particle in a box of side proportional to a, we put the formula of DN Constant $(2\sqrt{2})/\pi = (2\sqrt{6})/a^2$ and obtain a. Successively, we obtain all the Ramanujan recurring numbers. This is the my thesis. As with the solution of the Schrodinger equation for a particle in a box of side proportional to a, so the DN Constant also works with solutions of the Schrodinger equations of that type. Regarding the DN

Constant (Del Gaudio-Nardelli Constant) . For V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , we obtain: ((1/3*√2*a^3)/(4/3*π*(a/2)^3)) = (2√2)/π. In this case, the box is the octahedron and the point particle inside the box represents the sphere inscribed in the octahedron. So, as in the microcosm, so macrocosm.

From the following formulas:

$$c_1 = \int_0^{a/2} A\sqrt{\frac{2}{a}}\sin\left(\frac{\pi}{a}x\right)dx = \frac{2}{a}\int_0^{a/2}\sin\left(\frac{\pi}{a}x\right)dx$$

$$c_1 = \frac{2}{a}\frac{a}{\pi}\left[-\cos\left(\frac{\pi}{a}\frac{x}{2}\right)\right]_0^{a/2} = \frac{2}{\pi}\left[-\cos\left(\frac{\pi}{2}\right)+1\right] = \frac{2}{\pi}$$

$$P_1 = |c_1|^2 = \left(\frac{2}{\pi}\right)^2 = \frac{4}{\pi^2}$$

$$P_1 = 0.405$$

we have that:

if we consider $\sqrt{2}\ c_1$, we obtain:

$\sqrt{2}\ (2/\pi) = (2\sqrt{2})/\pi$ e $P_1 = (\sqrt{2}\ c_1)^2 = ((2\sqrt{2})/\pi)^2 = 8/\pi^2$

$((2\sqrt{2})/\pi)^2$

Input

$$\left(\frac{2\sqrt{2}}{\pi}\right)^2$$

Result

$$\frac{8}{\pi^2}$$

Decimal approximation

0.810569469138702171551035705677821111234870197377972390764872255 1

…

0.8105694691387…

Property

$\frac{8}{\pi^2}$ is a transcendental number

Series representations

$$\left(\frac{2\sqrt{2}}{\pi}\right)^2 = \frac{1}{2\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^2}$$

$$\left(\frac{2\sqrt{2}}{\pi}\right)^2 = \frac{1}{2\left(\sum_{k=0}^{\infty} \frac{(-1)^k\, 1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^2}$$

$$\left(\frac{2\sqrt{2}}{\pi}\right)^2 = \frac{8}{\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2}$$

Integral representations

$$\left(\frac{2\sqrt{2}}{\pi}\right)^2 = \frac{1}{2\left(\int_0^1 \sqrt{1-t^2}\ dt\right)^2}$$

$$\left(\frac{2\sqrt{2}}{\pi}\right)^2 = \frac{2}{\left(\int_0^\infty \frac{1}{1+t^2}\ dt\right)^2}$$

$$\left(\frac{2\sqrt{2}}{\pi}\right)^2 = \frac{2}{\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\ dt\right)^2}$$

Basically the previous result 0.405 multiplied by 2.

In fact:

$$2 \cdot \frac{4}{\pi^2}$$

2·4/π^2

Input

$$2 \times \frac{4}{\pi^2}$$

Result

$$\frac{8}{\pi^2}$$

Decimal approximation

0.810569469138702171551035705677821111234870197377972390764872255 1

…

0.8105694691387…

Property

$\frac{8}{\pi^2}$ is a transcendental number

Alternative representations

$$\frac{2\times 4}{\pi^2} = \frac{8}{(180^\circ)^2}$$

$$\frac{2\times 4}{\pi^2} = \frac{8}{6\zeta(2)}$$

$$\frac{2\times 4}{\pi^2} = \frac{8}{(-i\log(-1))^2}$$

$\zeta(s)$ is the Riemann zeta function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Series representations

$$\frac{2\times 4}{\pi^2} = \frac{1}{2\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2}$$

$$\frac{2\times 4}{\pi^2} = \frac{1}{2\left(\sum_{k=0}^{\infty} \frac{(-1)^k\, 1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^2}$$

$$\frac{2\times 4}{\pi^2} = \frac{8}{\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2}$$

Integral representations

$$\frac{2\times 4}{\pi^2} = \frac{1}{2\left(\int_0^1 \sqrt{1-t^2}\; dt\right)^2}$$

$$\frac{2\times 4}{\pi^2} = \frac{2}{\left(\int_0^{\infty} \frac{1}{1+t^2}\, dt\right)^2}$$

$$\frac{2\times 4}{\pi^2} = \frac{2}{\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\, dt\right)^2}$$

From which, after some calculations:

((((1+((π^2/6 – 1)^1/32))((2·4)/π^2))))

Input

$$\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)\times\frac{2\times 4}{\pi^2}$$

Exact result

$$\frac{8\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)}{\pi^2}$$

Decimal approximation

1.610104681510625584950988987851645303498690573060170544000112152 2

…

1.61010468151062… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate forms

$$\frac{8\left(1+\sqrt[32]{\frac{1}{6}\left(\pi^2-6\right)}\right)}{\pi^2}$$

$$\frac{4\left(6+6^{31/32}\sqrt[32]{\pi^2-6}\right)}{3\,\pi^2}$$

$$\frac{4\times 2^{31/32}\left(\sqrt[32]{6}+\sqrt[32]{\pi^2-6}\right)}{\sqrt[32]{3}\;\pi^2}$$

Expanded form

$$\frac{8}{\pi^2}+\frac{8\sqrt[32]{\frac{\pi^2}{6}-1}}{\pi^2}$$

Alternative representations

$$\frac{\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)(2\times 4)}{\pi^2} = \frac{8\left(1+\sqrt[32]{-1+\zeta(2)}\right)}{6\,\zeta(2)}$$

$$\frac{\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)(2\times 4)}{\pi^2} = \frac{8\left(1+\sqrt[32]{-1+\frac{1}{6}(180^\circ)^2}\right)}{(180^\circ)^2}$$

$$\frac{\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)(2\times 4)}{\pi^2} = \frac{8\left(1+\sqrt[32]{-1+\frac{1}{6}\cos^{-1}(-1)^2}\right)}{\cos^{-1}(-1)^2}$$

$\zeta(s)$ is the Riemann zeta function

$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$\frac{\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)(2\times 4)}{\pi^2} = \frac{8}{\pi^2}+\frac{8\sqrt[32]{-1+\sum_{k=1}^{\infty}\frac{1}{k^2}}}{\pi^2}$$

$$\frac{\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)(2\times 4)}{\pi^2} = \frac{8}{\pi^2}+\frac{8\sqrt[32]{-1-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}}}{\pi^2}$$

$$\frac{\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)(2\times 4)}{\pi^2}=\frac{8}{\pi^2}+\frac{8\sqrt[32]{-1+\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}}}{\pi^2}$$

Integral representations

$$\frac{\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)(2\times 4)}{\pi^2}=\frac{2\left(3+3^{31/32}\sqrt[32]{-3+2\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2}\right)}{3\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2}$$

$$\frac{\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)(2\times 4)}{\pi^2}=\frac{2\left(3+3^{31/32}\sqrt[32]{-3+2\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)^2}\right)}{3\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)^2}$$

$$\frac{\left(1+\sqrt[32]{\frac{\pi^2}{6}-1}\right)(2\times 4)}{\pi^2}=\frac{3+3^{31/32}\sqrt[32]{-3+8\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2}}{6\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2}$$

And again:

π/137+2((2·4)/π^2))

where 1/137 ≈ Fine Structure constant

Input

$$\frac{\pi}{137}+2\times\frac{2\times 4}{\pi^2}$$

Result

$$\frac{16}{\pi^2}+\frac{\pi}{137}$$

Decimal approximation

1.64407027151528604557260165502921523622300440497036161233978060 22

…

1.644070271515286…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{16}{\pi^2} + \frac{\pi}{137}$ is a transcendental number

Alternate form

$$\frac{\pi^3 + 2192}{137\,\pi^2}$$

Alternative representations

$$\frac{\pi}{137} + \frac{2\,(2\times 4)}{\pi^2} = \frac{180^\circ}{137} + \frac{16}{(180^\circ)^2}$$

$$\frac{\pi}{137} + \frac{2\,(2\times 4)}{\pi^2} = \frac{1}{137}\cos^{-1}(-1) + \frac{16}{\cos^{-1}(-1)^2}$$

$$\frac{\pi}{137} + \frac{2\,(2\times 4)}{\pi^2} = -\frac{1}{137}\,i\log(-1) + \frac{16}{(-i\log(-1))^2}$$

$\cos^{-1}(x)$ is the inverse cosine function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Series representations

$$\frac{\pi}{137}+\frac{2\,(2\times 4)}{\pi^2}=\frac{137+4\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^3}{137\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2}$$

$$\frac{\pi}{137}+\frac{2\,(2\times 4)}{\pi^2}=\frac{2192+\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^3}{137\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2}$$

$$\frac{\pi}{137}+\frac{2\,(2\times 4)}{\pi^2}=\frac{2192+\left(\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{-5-8k}-\frac{1}{2+4k}+\frac{4}{1+8k}-\frac{1}{6+8k}\right)\right)^3}{137\left(\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{-5-8k}-\frac{1}{2+4k}+\frac{4}{1+8k}-\frac{1}{6+8k}\right)\right)^2}$$

Integral representations

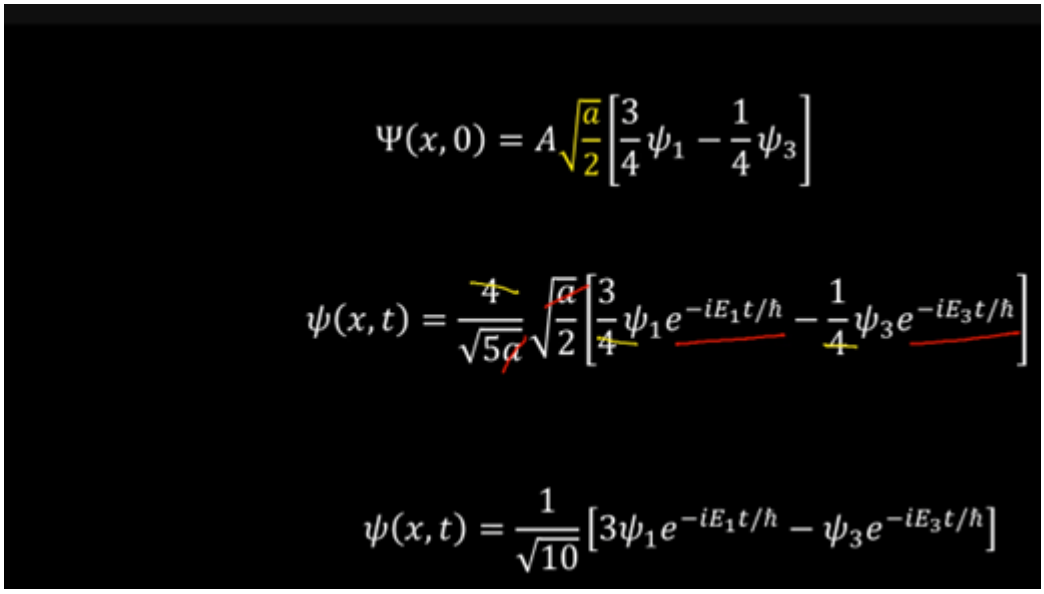
$$\frac{\pi}{137}+\frac{2\,(2\times 4)}{\pi^2}=\frac{2\left(274+\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^3\right)}{137\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2}$$

$$\frac{\pi}{137}+\frac{2\,(2\times 4)}{\pi^2}=\frac{2\left(274+\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)^3\right)}{137\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)^2}$$

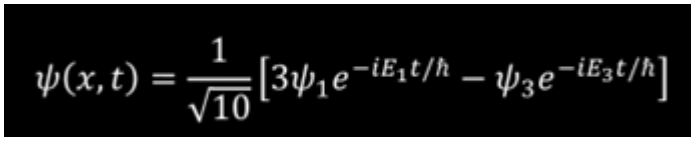
$$\frac{\pi}{137}+\frac{2\,(2\times 4)}{\pi^2}=\frac{2\left(274+\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^3\right)}{137\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^2}$$

From

Expectation value of time dependent wave function -
https://www.youtube.com/watch?v=K0gODNMd6bQ


$\Psi(x,0) = A\sqrt{\frac{a}{2}}\left[\frac{3}{4}\psi_1 - \frac{1}{4}\psi_3\right]$
$\psi(x,t) = \frac{4}{\sqrt{5a}}\sqrt{\frac{a}{2}}\left[\frac{3}{4}\psi_1 e^{-iE_1 t/\hbar} - \frac{1}{4}\psi_3 e^{-iE_3 t/\hbar}\right]$
$\psi(x,t) = \frac{1}{\sqrt{10}}\left[3\psi_1 e^{-iE_1 t/\hbar} - \psi_3 e^{-iE_3 t/\hbar}\right]$

Multiplying



by $\frac{2\sqrt{2}}{\pi}$ and simplifying:

(2sqrt2)/Pi*1/(sqrt10)*[3*ψ*e^(-(i*B*)t/h)-ψ*e^(-(i*B*t)/h)]

Input

$$\frac{2\sqrt{2}}{\pi} \times \frac{1}{\sqrt{10}}\left(3\,\psi\, e^{-(i\,B)^* \times t/\hbar} - \psi\, e^{-(i\,B\,t)/\hbar}\right)$$

z^* is the complex conjugate of z

i is the imaginary unit

Exact result

$$\frac{2\left(3\,\psi\,e^{(i\,t\,B^*)/h}-\psi\,e^{-(i\,B\,t)/h}\right)}{\sqrt{5}\,\pi}$$

Alternate forms assuming B, h, t, and ϕ are real

$$\frac{4\,\psi\left(\cos\left(\frac{B\,t}{h}\right)+2\,i\,\sin\left(\frac{B\,t}{h}\right)\right)}{\sqrt{5}\,\pi}$$

$$\frac{4\,\psi\cos\left(\frac{B\,t}{h}\right)}{\sqrt{5}\,\pi}+\frac{8\,i\,\psi\sin\left(\frac{B\,t}{h}\right)}{\sqrt{5}\,\pi}$$

Alternate forms

$$\frac{2\,\psi\,e^{-(i\,B\,t)/h}\left(-1+3\,e^{(2\,i\,t\,\mathrm{Re}(B))/h}\right)}{\sqrt{5}\,\pi}$$

$$\psi\left(\frac{6\,e^{(i\,t\,B^*)/h}}{\sqrt{5}\,\pi}-\frac{2\,e^{-(i\,B\,t)/h}}{\sqrt{5}\,\pi}\right)$$

$$\frac{2\,\psi\,e^{-(i\,B\,t)/h}\left(-1+3\,e^{(i\,t\,B^*)/h+(i\,B\,t)/h}\right)}{\sqrt{5}\,\pi}$$

$\mathrm{Re}(z)$ is the real part of z

Expanded form

$$\frac{6\,\psi\,e^{(i\,t\,B^*)/h}}{\sqrt{5}\,\pi}-\frac{2\,\psi\,e^{-(i\,B\,t)/h}}{\sqrt{5}\,\pi}$$

Alternate forms assuming B, h, t, and ϕ are positive

$$\frac{2\psi\, e^{-(i\,Bt)/h}\left(-1+3\,e^{(2\,i\,Bt)/h}\right)}{\sqrt{5}\ \pi}$$

$$\frac{6\psi\, e^{(i\,Bt)/h}}{\sqrt{5}\ \pi}-\frac{2\psi\, e^{-(i\,Bt)/h}}{\sqrt{5}\ \pi}$$

Roots

$$\mathrm{Re}(B)\neq 0\,,\quad \mathrm{Re}(h)>0\,,\quad t=\frac{h\,(2\,\pi\,n+i\log(3))}{B^*+B}\,,\quad n\in\mathbb{Z}$$

$$\mathrm{Re}(B)\neq 0\,,\quad \mathrm{Re}(h)<0\,,\quad t=\frac{h\,(2\,\pi\,n+i\log(3))}{B^*+B}\,,\quad n\in\mathbb{Z}$$

$$\mathrm{Re}(B)\neq 0\,,\quad \mathrm{Im}(h)\neq 0\,,\quad \mathrm{Re}(h)=0\,,\quad t=\frac{h\,(2\,\pi\,n+i\log(3))}{B^*+B}\,,\quad n\in\mathbb{Z}$$

$\log(x)$ is the natural logarithm

$\mathbb{Z}$ is the set of integers

$\mathrm{Im}(z)$ is the imaginary part of z

Root

$$\psi=0$$

Series expansion at t=0

$$\frac{4\,\psi}{\sqrt{5}\ \pi}+\frac{2\,i\,t\,\psi\,(3\,B^*+B)}{\sqrt{5}\ \pi\,h}+\frac{t^2\,\psi\left(B^2-3\,(B^*)^2\right)}{\sqrt{5}\ \pi\,h^2}-\frac{i\,t^3\,\psi\left(B^3+3\,(B^*)^3\right)}{3\sqrt{5}\ \pi\,h^3}-\frac{t^4\left(\psi\left(B^4-3\,(B^*)^4\right)\right)}{12\left(\sqrt{5}\ \pi\,h^4\right)}+O\left(t^5\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial t}\left(\frac{\left(2\sqrt{2}\right)\left(3\psi\, e^{-\left((i\,B\right)^{*}\,t\right)/\hbar}-\psi\, e^{-(i\,B\,t)/\hbar}\right)}{\pi\sqrt{10}}\right)=\frac{2\,i\,\psi\, e^{-(i\,B\,t)/\hbar}\left(B+3\,B^{*}\,e^{\left(i\,t\left(B^{*}+B\right)\right)/\hbar}\right)}{\sqrt{5}\,\pi\,\hbar}$$

(assuming a function from reals to reals)

Indefinite integral

$$\int\frac{2\left(-e^{-(i\,B\,t)/\hbar}\,\psi+3\,e^{\left(i\,t\,B^{*}\right)/\hbar}\,\psi\right)}{\sqrt{5}\,\pi}\,dt=$$
$$-\frac{2\,i\,\hbar\,\psi\left(\frac{e^{-(i\,B\,t)/\hbar}}{B}-\frac{3\,e^{-(i\,t\,(B-2\,\mathrm{Re}(B)))/\hbar}}{B-2\,\mathrm{Re}(B)}\right)}{\sqrt{5}\,\pi}+\text{constant}$$

Alternative representations

$$\frac{\left(3\psi\, e^{-\left((i\,B\right)^{*}\,t\right)/\hbar}-\psi\, e^{-(i\,B\,t)/\hbar}\right)\left(2\sqrt{2}\right)}{\sqrt{10}\,\pi}=\frac{\left(3\psi\, w^{a}-\psi\, w^{a}\right)\left(2\sqrt{2}\right)}{\sqrt{10}\,\pi}$$
$$\text{for}\left(a=-\frac{i\,B\,t}{\hbar\log(w)}\text{ and }a=\frac{i\,t\,B^{*}}{\hbar\log(w)}\right)$$

$$\frac{\left(3\psi\, e^{-\left((i\,B\right)^{*}\,t\right)/\hbar}-\psi\, e^{-(i\,B\,t)/\hbar}\right)\left(2\sqrt{2}\right)}{\sqrt{10}\,\pi}=\frac{2\left(-\psi\, e^{-(i\,(B\,t))/\hbar}+3\,\psi\, e^{-\left(t\,|B\,i|^{2}\right)/(\hbar\,(B\,i))}\right)\sqrt{2}}{\pi\sqrt{10}}$$

$$\frac{\left(3\psi\, e^{-\left((i\,B\right)^{*}\,t\right)/\hbar}-\psi\, e^{-(i\,B\,t)/\hbar}\right)\left(2\sqrt{2}\right)}{\sqrt{10}\,\pi}=$$
$$\frac{2\left(-\psi\, e^{-(i\,(B\,t))/\hbar}+3\,\psi\, e^{-(i\,(B\,t))/\left(\hbar\,\mathrm{sgn}(B\,i)^{2}\right)}\right)\sqrt{2}}{\pi\sqrt{10}}$$

$|z|$ is the absolute value of z

$\mathrm{sgn}(x)$ is the sign of x

Series representations

$$\frac{\left(3\psi\, e^{-((i\,B)^*\,t)/\hbar}-\psi\, e^{-(i\,B\,t)/\hbar}\right)\left(2\sqrt{2}\right)}{\sqrt{10}\,\pi}=\sum_{k=0}^{\infty}-\frac{2\psi\left(\left(-\frac{i\,B\,t}{\hbar}\right)^k-3\left(\frac{i\,t\,B^*}{\hbar}\right)^k\right)}{\sqrt{5}\,\pi\,k!}$$

$$\frac{\left(3\psi\, e^{-((i\,B)^*\,t)/\hbar}-\psi\, e^{-(i\,B\,t)/\hbar}\right)\left(2\sqrt{2}\right)}{\sqrt{10}\,\pi}=\sum_{k=0}^{\infty}-\frac{2\,e^{z_0}\,\psi\left(\left(-\frac{i\,B\,t}{\hbar}-z_0\right)^k-3\left(\frac{i\,t\,B^*}{\hbar}-z_0\right)^k\right)}{\sqrt{5}\,\pi\,k!}$$

$$\frac{\left(3\psi\, e^{-((i\,B)^*\,t)/\hbar}-\psi\, e^{-(i\,B\,t)/\hbar}\right)\left(2\sqrt{2}\right)}{\sqrt{10}\,\pi}=$$
$$\sum_{k=0}^{\infty}\frac{2\psi\left(-\left(-\frac{i\,B\,t}{\hbar}\right)^{2\,k}(h+2\,h\,k-i\,B\,t)+3\left(\frac{i\,t\,B^*}{\hbar}\right)^{2k}(h+2\,h\,k+i\,t\,B^*)\right)}{\sqrt{5}\,h\,\pi\,(1+2\,k)!}$$

$n!$ is the factorial function

From the following alternate form

$$\frac{4\,\psi\cos\left(\frac{B\,t}{\hbar}\right)}{\sqrt{5}\,\pi}+\frac{8\,i\,\psi\sin\left(\frac{B\,t}{\hbar}\right)}{\sqrt{5}\,\pi}$$

for B = 2 and t = 8:

(4 ψ cos((2*8)/(1.054571e-34)))/(sqrt(5) π) + (8 i ψ sin((2*8)/(1.054571e-34)))/(sqrt(5) π)

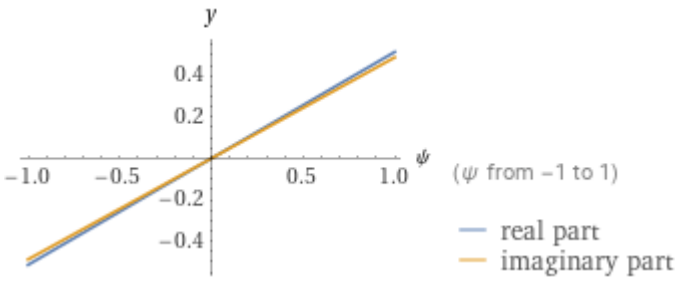
Input interpretation

$$\frac{4\,\psi\cos\left(\frac{2\times8}{1.054571\times10^{-34}}\right)}{\sqrt{5}\,\pi}+\frac{8\,i\,\psi\sin\left(\frac{2\times8}{1.054571\times10^{-34}}\right)}{\sqrt{5}\,\pi}$$

Result

$(0.514431 + 0.488213\,i)\,\psi$

Plot



Alternate form assuming ϕ is real

$i\,(0.488213\,\psi + 0) + 0.514431\,\psi + 0$

Property as a function

Parity

odd

Derivative

$$\frac{d}{d\psi}((0.514431 + 0.488213\,i)\,\psi) = 0.514431 + 0.488213\,i$$

Indefinite integral

$$\int (0.514431 + 0.488213\,i)\,\psi\,d\psi = (0.257216 + 0.244107\,i)\,\psi^2 + \text{constant}$$

From which, for ψ = 1:

(4 cos((2*8)/(1.054571e-34)))/(sqrt(5) π) + (8 i sin((2*8)/(1.054571e-34)))/(sqrt(5) π)

Input interpretation

$$\frac{4\cos\left(\frac{2\times 8}{1.054571\times 10^{-34}}\right)}{\sqrt{5}\,\pi}+\frac{8\,i\sin\left(\frac{2\times 8}{1.054571\times 10^{-34}}\right)}{\sqrt{5}\,\pi}$$

Result

0.540725… +
0.356899… i

Alternate complex forms

$0.514431 + 0.488213\,i$

$0.709219\,(\cos(0.759255) + i\sin(0.759255))$

$0.709219\,e^{0.759255\,i}$

Polar coordinates

$r = 0.709219$ (radius), $\theta = 0.759255$ (angle)

0.709219

And, after some calculations:

(Pi-√3/2)*(((4 cos((2*8)/(1.054571e-34)))/(sqrt(5) π) + (8 i sin((2*8)/(1.054571e-34)))/(sqrt(5) π)))

Input interpretation

$$\left(\pi - \frac{\sqrt{3}}{2}\right)\left(\frac{4\cos\left(\frac{2\times 8}{1.054571\times 10^{-34}}\right)}{\sqrt{5}\,\pi} + \frac{8\,i\sin\left(\frac{2\times 8}{1.054571\times 10^{-34}}\right)}{\sqrt{5}\,\pi}\right)$$

Result

1.23046… +
0.812148… i

Alternate complex forms

$1.17062 + 1.11096\,i$

$1.61388\,(\cos(0.759255) + i\sin(0.759255))$

$1.61388\,e^{0.759255\,i}$

Polar coordinates

$r = 1.61388$ (radius), $\theta = 0.759255$ (angle)

1.61388 result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We have also:

(16/(π^4))(1/4(9*1/((4 cos((2*8)/(1.054571e-34)))/(sqrt(5) π) + (8 i sin((2*8)/(1.054571e-34)))/(sqrt(5) π))))^2

Input interpretation

$$\frac{16}{\pi^4}\left(\frac{1}{4}\left(9\times\frac{1}{\frac{4\cos\left(\frac{2\times 8}{1.054571\times 10^{-34}}\right)}{\sqrt{5}\ \pi}+\frac{8\,i\sin\left(\frac{2\times 8}{1.054571\times 10^{-34}}\right)}{\sqrt{5}\ \pi}}\right)\right)^2$$

i is the imaginary unit

Result

0.778723647158700813223845866868750553086526768755047154124229482…

− 1.821521385596101781119586325887075652032212373829176803564338 27… i

Alternate complex forms

0.086399 − 1.65094 i

$1.6532\,(\cos(-1.51851)+i\sin(-1.51851))$

$1.6532\,e^{-1.51851\,i}$

Polar coordinates

$r = 1.6532$ (radius), $\theta = -1.51851$ (angle)

1.6532 result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3$ = 1164.2696 i.e. 1.6557845...

Indeed:

$$G_{505} = P^{-1/4}Q^{1/6} = (\sqrt{5}+2)^{1/2}\left(\frac{\sqrt{5}+1}{2}\right)^{1/4}(\sqrt{101}+10)^{1/4} \times \left((130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}}\right)^{1/6}.$$

Thus, it remains to show that

$$(130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}} = \left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3,$$

which is straightforward. □

We observe that:

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3 = 1164.2696\ldots$$

Indeed:

Input

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3$$

Result

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3$$

Decimal approximation

1164.269601267364667589866974010779128760584499596965142888579477 7

…

1164.269601267….

Alternate forms

$$\frac{1}{64}\left(\sqrt{10\left(21+\sqrt{505}\right)}+\sqrt{2\left(113+5\sqrt{505}\right)}\right)^3$$

$$\frac{1}{64}\left(5\sqrt{5}+\sqrt{101}+\sqrt{105-40\,i}+\sqrt{105+40\,i}\right)^3$$

$$\sqrt{338\,881+15\,080\sqrt{505}+4\sqrt{5\left(2\,871\,007\,052+127\,758\,137\sqrt{505}\right)}}$$

Expanded form

$$\frac{111}{4}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{5}{4}\sqrt{\frac{505}{2}\left(105+5\sqrt{505}\right)}+\frac{107}{4}$$
$$\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}+\frac{5}{4}\sqrt{\frac{505}{2}\left(113+5\sqrt{505}\right)}$$

Minimal polynomial

$$x^8-1\,355\,524\,x^6+400\,646\,x^4-1\,355\,524\,x^2+1$$

From which, we obtain:

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3}=1{,}65578\ldots$$

Indeed:

Input

$$\sqrt[14]{\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3}$$

Result

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^{3/14}$$

Decimal approximation

1.655784548804744724619349561761107639558068114480697960239356794 4

…

1.6557845488….

Alternate forms

$$\left(\frac{1}{4}\left(\sqrt{10\left(21+\sqrt{505}\right)}+\sqrt{2\left(113+5\sqrt{505}\right)}\right)\right)^{3/14}$$

$$\sqrt[28]{338\,881 + 15\,080\sqrt{505} + 4\sqrt{5\left(2\,871\,007\,052 + 127\,758\,137\sqrt{505}\right)}}$$

$$\frac{\left(5\sqrt{5} + \sqrt{101} + \sqrt{105 - 40\,i} + \sqrt{105 + 40\,i}\right)^{3/14}}{2^{3/7}}$$

Minimal polynomial

$$x^{112} - 1\,355\,524\,x^{84} + 400\,646\,x^{56} - 1\,355\,524\,x^{28} + 1$$

All 14th roots of (1/2 sqrt(1/2 (105+5 sqrt(505)))+1/2 sqrt(1/2 (113+5 sqrt(505))))^3

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105 + 5\sqrt{505}\right)} + \frac{1}{2}\sqrt{\frac{1}{2}\left(113 + 5\sqrt{505}\right)}\right)^{3/14} e^{0} \approx 1.65578$$ (real, principal root)

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105 + 5\sqrt{505}\right)} + \frac{1}{2}\sqrt{\frac{1}{2}\left(113 + 5\sqrt{505}\right)}\right)^{3/14} e^{(i\pi)/7} \approx 1.4918 + 0.7184\,i$$

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105 + 5\sqrt{505}\right)} + \frac{1}{2}\sqrt{\frac{1}{2}\left(113 + 5\sqrt{505}\right)}\right)^{3/14} e^{(2i\pi)/7} \approx 1.0324 + 1.2945\,i$$

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105 + 5\sqrt{505}\right)} + \frac{1}{2}\sqrt{\frac{1}{2}\left(113 + 5\sqrt{505}\right)}\right)^{3/14} e^{(3i\pi)/7} \approx 0.36845 + 1.61427\,i$$

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105 + 5\sqrt{505}\right)} + \frac{1}{2}\sqrt{\frac{1}{2}\left(113 + 5\sqrt{505}\right)}\right)^{3/14} e^{(4i\pi)/7} \approx -0.3684 + 1.61427\,i$$

1/2((1.6532 + 1.61388)/2 + 1.6532)

Input interpretation

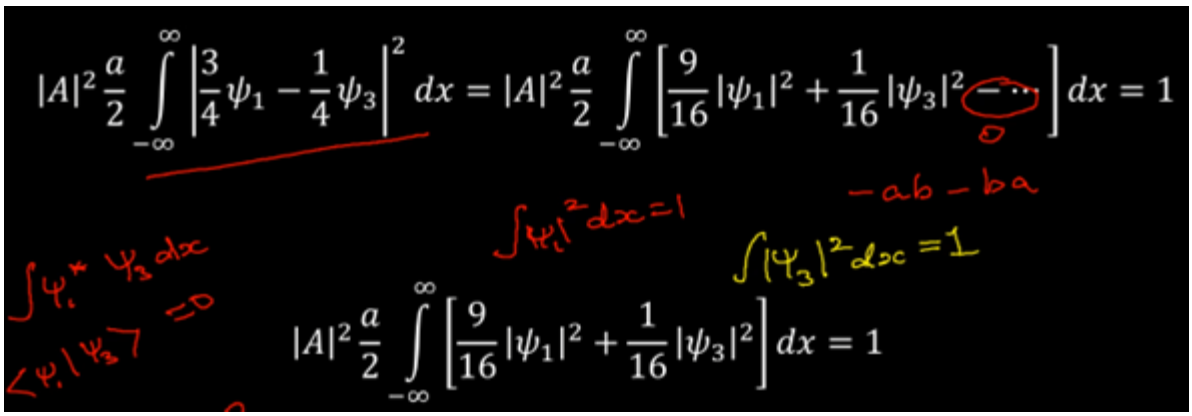
$$\frac{1}{2}\left(\frac{1.6532 + 1.61388}{2} + 1.6532\right)$$

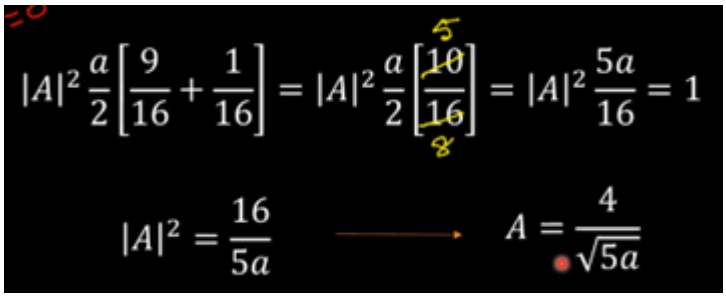
Result

1.64337

1.64337 ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

From


$|A|^2 \frac{a}{2} \int_{-\infty}^{\infty} \left|\frac{3}{4}\psi_1 - \frac{1}{4}\psi_3\right|^2 dx = |A|^2 \frac{a}{2} \int_{-\infty}^{\infty} \left[\frac{9}{16}|\psi_1|^2 + \frac{1}{16}|\psi_3|^2 - \cdots\right] dx = 1$
$\int \psi_1^* \psi_3 dx$
$\langle \psi_1 | \psi_3 \rangle = 0$
$\int |\psi_1|^2 dx = 1$
$-ab - ba$
$\int |\psi_3|^2 dx = 1$
$|A|^2 \frac{a}{2} \int_{-\infty}^{\infty} \left[\frac{9}{16}|\psi_1|^2 + \frac{1}{16}|\psi_3|^2\right] dx = 1$


$|A|^2 \frac{a}{2}\left[\frac{9}{16} + \frac{1}{16}\right] = |A|^2 \frac{a}{2}\left[\frac{10}{16}\right] = |A|^2 \frac{5a}{16} = 1$
5
8
$|A|^2 = \frac{16}{5a} \longrightarrow A = \frac{4}{\sqrt{5a}}$

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

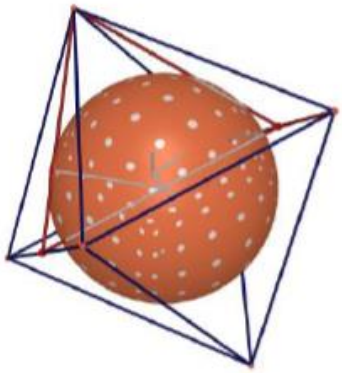


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\ 2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

$0.900316316157106\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2=\frac{2}{3}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2=\frac{2}{3}\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ $\zeta(2) = \pi^2/6 = 1.644934$ (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

$2\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number

1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^8\right)^{-k}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^8\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+\right.$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+\right.$$
$$27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$

$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^{\,8}-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^{\,8}-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π²

$\pi e^{0} \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4(-1)^k\, 1195^{-1-2k}\left(5^{1+2k} - 4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1 \sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

$0.900316316157106\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003 \quad \text{(real, principal root)}$$

$$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003 \quad \text{(real root)}$$

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{8}{\pi^2}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

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i is the imaginary unit

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{(15+5\sqrt{5})\,\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)3}}{3\times3}} = \sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)3}}{3\times3}} = \sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))) = 3.1415926535

Indeed:

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535…. = π (Ramanujan Recurring Number)

From the following extended DN Constant (“Unitary Formula”)

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

with regard

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

for q = 1729 and p = 4096, we obtain by changing the sign in the algebraic sum of the aforementioned Cardano's Formula and after some calculations:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

multiplied by

$$\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}$$

and performing the ninth root of the entire expression:

√((2^(-1/π)(5(3+√(5))π)^(1/(2π)))((∛(-1729/2+√(1729^2/4+4096^3/27)) -∛(-1729/2-√(1729^2/4+4096^3/27)))^1/9))

we obtain:

$$\sqrt{\left(2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}}$$

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}-\frac{1729}{2}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\sqrt[4\pi]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/(4π))

= 1.61549140391….

The general "unitary" formula, which derives from DN Constant, is the following:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

Where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

C is any constant or solution to an equation, R is the radius of the Universe and $2.33*10^{-13}$ is the temperature of the universe expressed in GeV.

For example, $C = 9.9128$, inserting a radius of the Universe, which we have calculated, equal to $R = 2.06274*10^{12}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}$$

1.6180359123482642354401744088347098542545273508401733563064818107
…

1.618035912348…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We obtain also:
(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))dxdydz

Indefinite integral assuming all variables are real

$0.809018\, x^2\, y\, z$ + constant

Definite integral over a cube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} 1.61804\, dx\, dy\, dz = 12.9443\, L^3$$

Definite integral over a sphere of radius R

$$\iiint_{x^2+y^2+z^2<R^2} 1.61804\, dz\, dy\, dx = 6.77761\, R^3$$

From which, for L = R = 1 , dividing the two definite integral results by the original expression, we obtain:

12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))

Input interpretation

$$\frac{12.9443}{\sqrt{2\times\frac{\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

8.00001…

8.00001…. ≈ 8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

And

3*(6.77761/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))

Input interpretation

$$3\times\frac{6.77761}{\sqrt{2\times\frac{\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

12.5664…

12.5664…. ≈ 4π = Bekenstein-Hawking (S_{BH}) black hole entropy

New fundamental formula deriving from DN Constant

The DN Constant (Del Gaudio-Nardelli Constant) equals $(2\sqrt{2})/\pi$ is defined as the ratio of the volume of an octahedron to the volume of a sphere and is an intriguing mathematical concept. Michele Nardelli hypothesized that the regular octahedron represents a phase in which the universe is highly symmetrical and with very low entropy. On the other hand, the sphere (which is inscribed in the octahedron, i.e. is "inside" it) represents the universe emerging from the quantum vacuum, which over time increases entropy and undergoes various symmetry breakings. This occurs in a regime of eternal inflation.

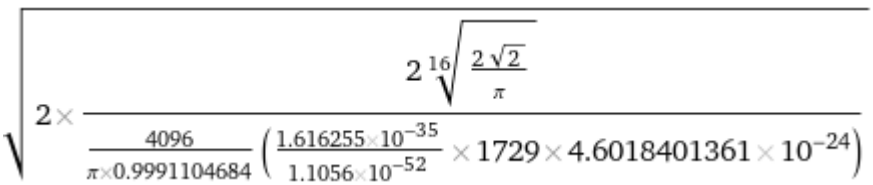
From the following expression

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×C×R)))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\,C\,R\right)}}$$

which comes from the DN Constant, with $1.616255*10^{-35}$ which is equal to the Planck length, $1.1056*10^{-52}$ which is equal to the Cosmological Constant, C = 1729 which corresponds to the Hardy-Ramanujan number and R = $4.6018401361 \times 10^{-24}$, which represents the radius of the Universe, we obtain:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×4.6018401361 × 10^-24))) = 1.6180329973...



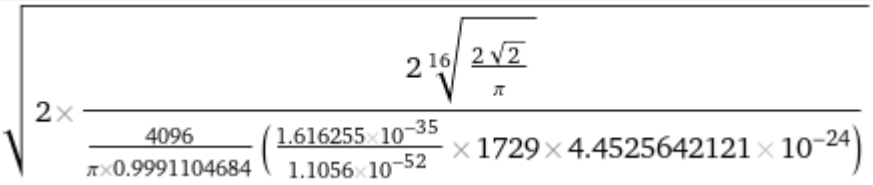
1.6180329973075324915067570166297467464225772608929671407919395903
…

1.6180329973075… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We have also the following formula:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(4.4525642121 × 10^-24))))

Input interpretation



Result

1.644932352102092130483898983704151168876621841655166877914166033 8
…

1.64493235210209213…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

And again:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(1.2206935225 × 10^-24))))

Input interpretation

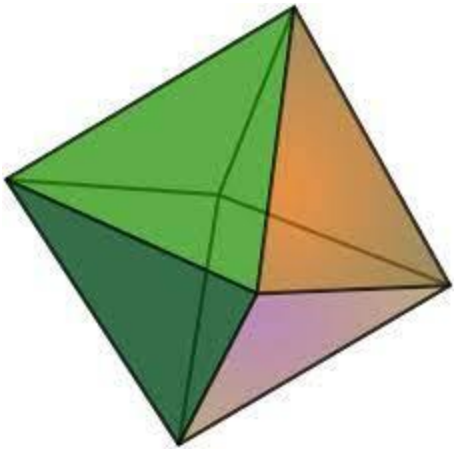
$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 1.2206935225\times 10^{-24}\right)}}$$

Result

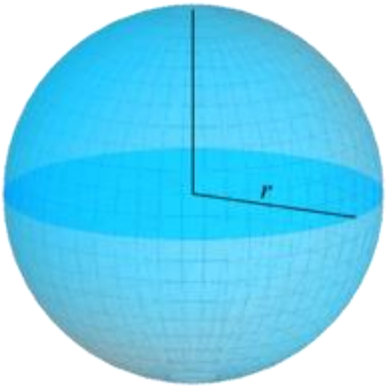
3.14159225731469939399500392316057964697311717816444239557900247 97…

3.14159225731469…≈ π (Ramanujan Recurring Number)

Now, we have that:



Octahedron



Sphere

Given the value of a volume, independently of the solid, following the Poincaré Conjecture, we compare any solid "without holes" and a sphere. If we compare an octahedron with a sphere, we have:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

If we consider the radius of the sphere as an unknown, we must find the value of the side of the octahedron which allows us to equalize the two volumes and which will give us the DN Constant as a result (which will therefore be equal to the radius of the sphere).

From

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

To find r we perform the following calculation:

$$r^3 = \frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi} = \frac{1}{3}\sqrt{2}a^3 \cdot \frac{3}{4\pi} = \frac{\sqrt{2}a^3}{4\pi} = \frac{\sqrt{2}\cdot\sqrt{2}a^3}{\sqrt{2}\cdot 4\pi} = \frac{2a^3}{\sqrt{2}\cdot 4\pi} = \frac{a^3}{2\sqrt{2}\pi}$$

$$r^3 = \frac{a^3}{2\sqrt{2}\pi}; \qquad r = \sqrt[3]{\frac{a^3}{2\sqrt{2}\pi}} = \frac{a}{\sqrt[3]{2\sqrt{2}\pi}}$$

To find *a*, we have, for

$$r = \frac{2\sqrt{2}}{\pi}; \qquad \frac{a}{\sqrt[3]{2\sqrt{2}\pi}} = \frac{2\sqrt{2}}{\pi};$$

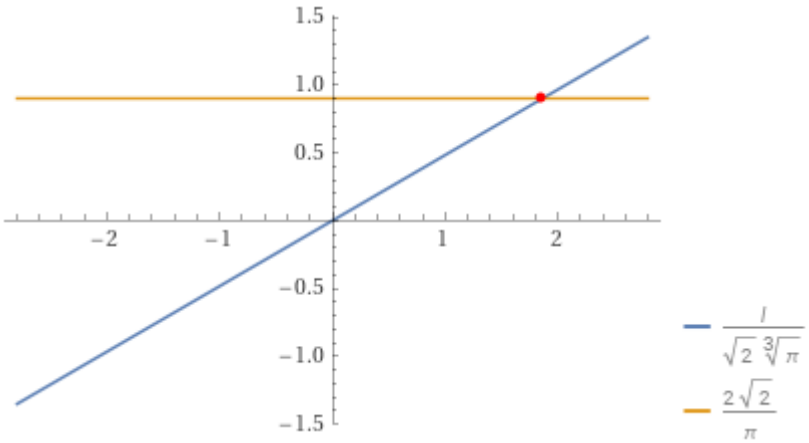
Thus, multiplying both the sides by $\sqrt[3]{2\sqrt{2}\pi}$, we obtain:

$$a = \frac{2\sqrt{2}}{\pi} \cdot \sqrt[3]{2\sqrt{2}\pi} = \sqrt[3]{\frac{(2\sqrt{2})^3 2\sqrt{2}\pi}{\pi^3}} =$$

$$= \sqrt[3]{\frac{8\cdot\sqrt{2^3}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{8\cdot\sqrt{2\cdot 2^2}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{16\cdot\sqrt{2}\cdot 2\sqrt{2}}{\pi^2}}$$

$$= \sqrt[3]{\frac{32\sqrt{2}\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{32 \cdot 2}{\pi^2}} = \frac{\sqrt[3]{64}}{\sqrt[3]{\pi^2}} = \frac{4}{\sqrt[3]{\pi^2}};\ a = \frac{4}{\sqrt[3]{\pi^2}}$$

Plot



Solution

$a = \frac{4}{\sqrt[3]{\pi^2}}$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), we obtain:

from the octahedron volume, we have: V = 1/3*√2*a^3 = (1/3*√2*($\frac{4}{\sqrt[3]{\pi^2}}$)^3)

(1/3*√2*(4/∛(π^2))^3)

Input

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Exact result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And, from the sphere volume V = (4/3*π*r^3) = (4/3*π*((2√2)/π)^3)

(4/3*π*((2√2)/π)^3)

Input

$$\frac{4}{3}\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3$$

Result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\sqrt{z_0}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\exp^3\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\left(\frac{1}{z_0}\right)^{3/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{3/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256x^8 + 95\,744x^7 - 3\,248\,750\,080x^6 - 914\,210\,725\,504x^5 + 15\,498\,355\,554\,921\,184x^4 + 2\,911\,478\,392\,539\,914\,656x^3 - 32\,941\,144\,911\,224\,677\,091\,680x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi =$$
$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256x^8+95\,744x^7-3\,248\,750\,080x^6-914\,210\,725\,504x^5+15\,498\,355\,554\,921\,184x^4+2\,911\,478\,392\,539\,914\,656x^3-32\,941\,144\,911\,224\,677\,091\,680x^2-3\,092\,528\,914\,069\,760\,354\,714\,456x+26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4}+\frac{29\sqrt{5}}{4}+54\sqrt{1093}-\frac{27}{8}\sqrt{10-2\sqrt{5}}-\frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

.

$$-\frac{107}{2}+\frac{\sqrt{5}}{2}+54\sqrt{1093}+\frac{27}{\sqrt{5}-1}-\frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

.

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

.

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

$1.64381856858\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2\left(\sqrt{5} - 1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256x^8 + 95\,744x^7 - 3\,248\,750\,080x^6 - 914\,210\,725\,504x^5 + 15\,498\,355\,554\,921\,184x^4 + 2\,911\,478\,392\,539\,914\,656x^3 - 32\,941\,144\,911\,224\,677\,091\,680x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256x^{120} + 95\,744x^{105} - 3\,248\,750\,080x^{90} - 914\,210\,725\,504x^{75} + 15\,498\,355\,554\,921\,184x^{60} + 2\,911\,478\,392\,539\,914\,656x^{45} - 32\,941\,144\,911\,224\,677\,091\,680x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1 + \sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4}+\frac{29\sqrt{5}}{4}+54\sqrt{1093}-\frac{27}{8}\sqrt{10-2\sqrt{5}}-\frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^{0}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.64382 \text{ (real, principal root)}$$

.

$$e^{(2i\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.50170+0.6686\,i$$

.

$$e^{(4i\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

.

$$e^{(2i\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

.

$$e^{(8i\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

.

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$
$$\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.$$
$$\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\,\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\,\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - z_0)^k\,z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\,dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\;\phi\;e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\;\phi\;e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\left(0.0814845\,b+1\right)^2 M^2\right) = 0.054323\left(0.0814845\,b+1\right) M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\left(226\,802\,245+18\,480\,874\,b\right) M^2}$$

$$\frac{\partial b(M,V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874}+b}{M}$$

$$\frac{\partial M(b,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\left(226\,802\,245+18\,480\,874\,b\right)^2 M}$$

$$\frac{\partial M(b,V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245+18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M} = \frac{2\left(226\,802\,245+18\,480\,874\,b\right)^2 M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b} = \frac{36\,961\,748\left(226\,802\,245+18\,480\,874\,b\right) M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}\left(0.0814845\,b+1\right)^2 M^2\right\} = 0 \text{ at } (b,M) = (-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

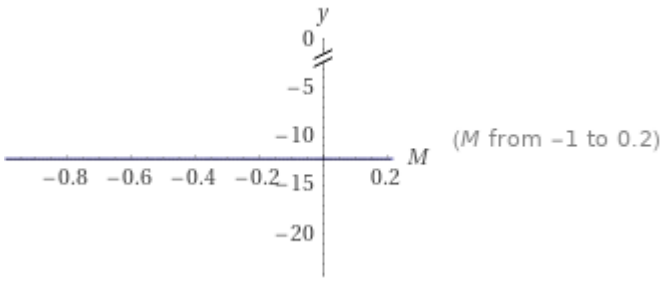
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

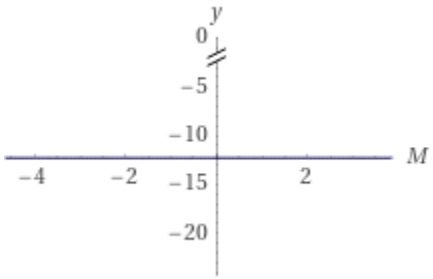
Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(M from −1 to 0.2)



(M from −4.6 to 3.9)

Alternate form assuming M is real:

−12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723\right)+O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

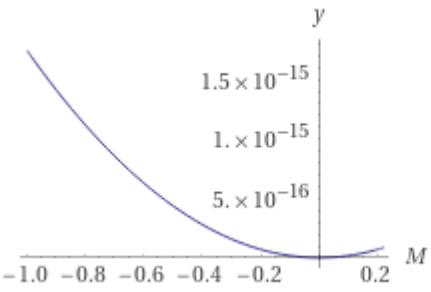
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

Result:

0

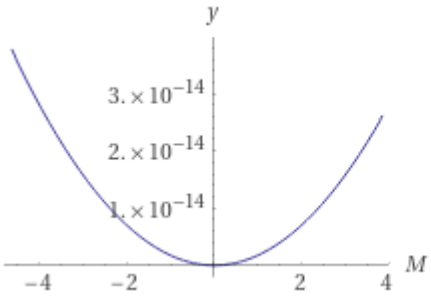
Plots: (possible mathematical connection with an open string)



(*M* from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

$M = 2 ; \; M = 3$

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O\left(M^{62\,194}\right)$

(Taylor series)

Series expansion at M = ∞:

$1.75541 \times 10^{-15} \, M^2 + O\left(\left(\frac{1}{M}\right)^{62\,194}\right)$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2(-0.5^2)$$

Result:

$$-4.3885134494746454534897078337808802083333333333333333333\ldots\times10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times10^{-17}$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 3^2+6.58545\times 10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times 3^2$$

Result:

$1.579864841810872363256294820161116875\times 10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-15}$

7.021621519*10^-15

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ; -4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

5.9776991059*10^-8 result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}))\ \hat{}\ (1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\ \sqrt[4]{3}\ e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182...

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

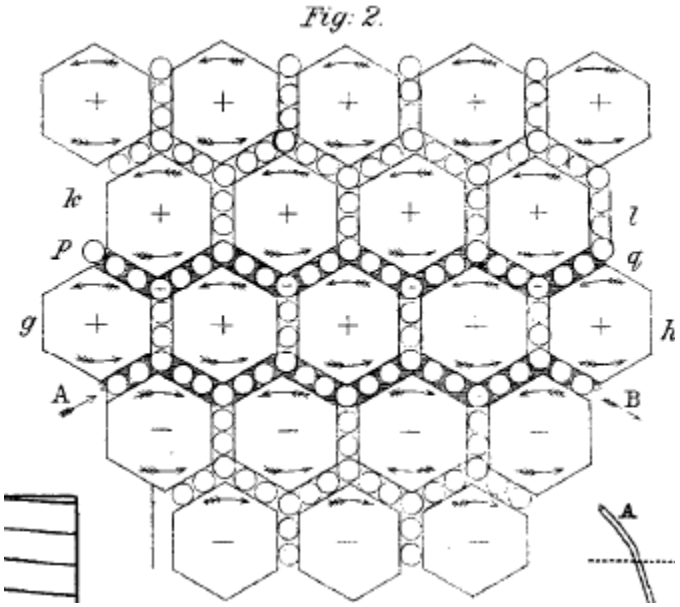
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

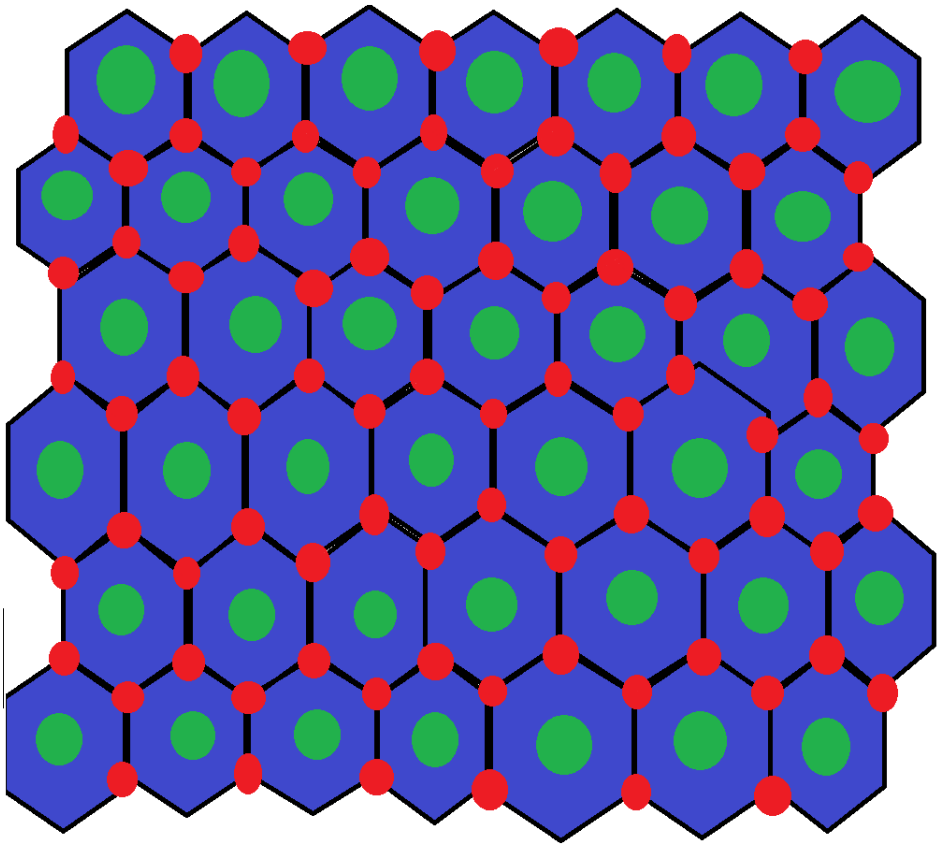
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability.

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An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

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